THE CIRCLES OF PROPORTION AND THE HORIZONTALL INSTRVMENT.

Both invented, and the vses of both

Written in Latine by

Mr. *W. O.*

Translated into English, and set forth for the publique benefit by

William Forster.

LONDON

Printed for Elias Allen *maker of these and all other Mathematical Instruments, and are to be sold at his shop over against S. Clements church without Temple-barre.*

1639.

TO

THE HONOVRABLE AND RENOWNED FOR

vertue, learning, and true valour,

Sir KENELME DIGBYE

Knight.

SIR,

THE excellent accompliſhments wherwith you are adorned both of vertue, and learning, and particularly in the Mathematicall Sciences, together with the Honourable reſpect the Author hereof beareth vnto your Worth, and his deſire to teſtifie the ſame, hath made mee preſume to preſent vnto you, and vnder the happy auſpice of your renowned name, to publiſh to the world this Treatiſe: the owning whereof though I may not chalenge to my ſelfe, yet the birth and production, whereby it hath a being to the benefit of others, is, as vnto a ſecond parent, due vnto me.

For being in the time of the long vacation 1630, in the Country, at the houſe of the Reue-

rend, and my moſt worthy friend, and Teacher, Mr William Oughtred (to whoſe inſtruction I owe both my initiation, and whole progreſſe in theſe Sciences.) Vpon occaſion of ſpeech told him of a Ruler of Numbers Sines, & Tangents, which one had beſpoken to be made (ſuch as is vſually called Mr. Gunters Ruler) 6 feet long, to be vſed with a payre of beame-compaſſes. "He an-"ſwered that was a poore invention, and the per-"formance very troubleſome: But, ſaid he, ſee-"ing you are taken with ſuch mechanicall wayes "of Inſtruments, I will ſhew you what deuiſes I "haue had by mee theſe many yeares. And firſt, hee brought to mee two Rulers of that ſort, to be vſed by applying one to the other, without any compaſſes: and after that hee ſhewed mee thoſe lines caſt into a circle or Ring, with another moueable circle vpon it. I ſeeing the great expeditenеſſe of both thoſe wayes; but eſpecially, of the latter. wherein it farre excelleth any other Inſtrument which hath bin knowne; told him, I wonder'd that hee could ſo many yeares conceale ſuch vſefull inuentions, not onely from the world, but from my ſelfe, to whom in other parts and myſteries of Art, he had bin ſo liberall. "He anſwered,

"That

"That the true way of Art is not by Instruments,
"but by Demonstration: and that it is a prepo-
"sterous course of vulgar Teachers, to begin
"with Instruments, and not with the Sciences,
"and so in-stead of Artists, to make their Schol-
"lers only doers of tricks, and as it were Iuglers:
"to the despite of Art, losse of precious time, and
"betraying of willing and industrious wits, vnto
"ignorance, and idlenesse. That the vse of Instru-
"ments is indeed excellent, if a man be an Artist:
"but contemptible, being set and opposed to Art.
"And lastly, that he meant to commend to me, the
"skill of Instruments, but first he would haue me
"well instructed in the Sciences. He also shewed
me many notes, and Rules for the vse of those
circles, and of his Horizontall Instrument,
(which he had proiected about 30 yeares before)
the most part written in Latine. All which I ob-
tained of him leaue to translate into English, and
make publique, for the vse, and benefit of such as
were studious, & louers of these excellent Sciences.

Which thing while I with mature, and diligent
care (as my occasions would giue me leaue) went
about to doe: another to whom the Author in a
louing confidence discouered this intent, vsing
more

more haſt then good ſpeed, went about to preocupate; of which vntimely birth, and preuenting (if not circumuenting) forwardneſſe, I ſay no more: but aduiſe the ſtudious Reader, onely ſo farre to truſt, as he ſhal be ſure doth agree to truth & Art.

And thus moſt noble Sir, without any brauing flouriſhes, or needleſſe multiplying of tautologized and erroneous præcepts, in naked truth, and in the modeſt ſimplicity, of the Author himſelfe (whoſe knowne skill in the whole Syſteme of Mathematicall learning, will eaſily free him from the ſuſpicion of hauing the way made for him, and the ſubiect vnuailed, to help his ſight) I haue notwithſtanding vnder the protection of your courteous fauour, and learned iudgement, perſiſted in my long conceiued purpoſe, of preſenting this tractate to the publique view, and light. Wiſhing withall vnto you encreaſe of deſerued honor, and happines.

May the 1. 1632.

By the honourer
and admirer
of your Worthines,

WILLIAM FORSTER.

TO THE ENGLISH GENTRIE,

and all others ſtudious of the MATHEMATICKS, which ſhall bee *Readers* hereof.

The juſt Apologie of WIL: OVGHTRED, againſt the ſlaunderous inſimulations of RICHARD DELAMAIN, in a Pamphlet called *Grammelogia*, or *the Mathematicall Ring*, or *Mirifica logarithmorum projectio circularis*.

HOnourable, and much honoured Gentlemen, I was of late at my comming up to London, for the performance of mine ordinary ſervice in the houſe of my moſt Honourable Lord the Earle of Arundell and Surrey, and Earle Marſhall of England, by many of my loving friends preſented with a moſt idle and ſcandalous Pamphlet written againſt me by *Richard Delamaine*, who profeſſeth himſelfe a Teacher of the Mathematickes about the City: Wherein I am brought before you upon the Scaffold, and with all the petulancies of a vexed mind and diſtempered paſſion, inſimulated and charged with, I know not what, injuries he pretendeth I ſhould have done unto him (your noble ſelves alſo by him ingaged therein, and incenſed againſt me) and at laſt, as if quite caſt, I am ſchooled by him with a long Lecture or Common place againſt *Slaunder and Detraction*. I did much wonder at it, to ſee my ſelfe ſo baſely and impudently abuſed by one whom I never had wronged, but had done very many courteſies for, giving him acceſſe to my chamber in Arundell Houſe day by day, teaching and inſtructing him in that facultie he profeſſeth: not onely ſatisfying

his ſcruples in thoſe things he partly knew: but even laying the very foundation of diverſe parts, whereof hee was utterly ignorant. And I did not ſo much marvell to ſee him ſo bold with me a poore man, but duſt and aſhes; as I was amazed to ſee him ſo fearefully (yet without feare) to play with Almighty God, hypocritically and againſt his owne conſcience in things apparantly falſe, invoking and challenging his all-knowing teſtimony: and in the middeſt of his moſt unmannerly raylings, in his booke; and his ſlaunderous backbiting and depraving me, by audacious intruding himſelfe upon my moſt honourable favours with falſe complaints, utterly to overthrow and diſcredite me; in a perſonated admonition againſt ſuch uncharitable calumniations, to pronounce judgement againſt himſelfe. But of theſe things we ſhall, God willing, ſee more in good time. I borrowed and peruſed that worthleſſe Pamphlet, and in reading it (I beſhrew him for making me caſt away ſo much of that little time is remayning to my declined yeares) I met with ſuch a patchery and confuſion of disjoynted ſtaffe, that I was ſtriken with a new wonder, that any man ſhould be ſo ſimple, as to ſhame himſelfe to the world with ſuch a hotch-potch.

In the two firſt pages, (for ſo he afterward calleth them) are two Schemes of his Inſtrument. In the fourth page is his Epiſtle to the Kings Majeſtie. In the 5, 6, 7, are verſes to his great commendation. In the 8 to the end of 21, he hath an Epiſtle opprobrious againſt me, moſt plainely ſtill pointing me out, that he needeth not to name me: and therein moſt learnedly diſputeth with me *his jealous oppoſite, and the ſuppoſed, and aſſumed author, and divulger*, and what not, in ſixe whole leaves, a queſtion about the aſſes ſhadow, I ſhould have ſayd, *whether the ring, or the Index at the center, bee the better?* that word BETTER cruelly wrings him. What, *ſuch a compariſon of* BETTER? *ſuch a comparative aſperſion of* BETTER? *Too great, and too looſe an aſperſion: An unſavory report indeed: which ſavours of too high a conceipt of the one, and too great a detraction from the other: Endeavouring, what in him lyeth, to annihilate and beate downe the way, which I write upon, and to glory in the rayſing up of his ſuppoſed owne: thereby not onely poſſeſſing men with an untruth, but making* ME *alſo ignorant in* MY *choyce, that* I *ſhould leave unto the world the weakeſt and imperfecteſt part of the projection of the Logarithmes, and leave the beſt for another to write upon. I never thought, when I firſt writ upon this my invention or my name ſo to come to the worlds rumour: which may teach* ME *and others carefulneſſe hereafter* (yea and fit it ſhould) *how and what* WE *publiſh to the world: ſeeing there are ſuch carpers and maligners, ſuch buſy-bodies, who marre what others make: ſuch who have ſtings like Bees, and arrowes*

rowes to ſhoote: *ſharp-witted Critiches, Diogenes-like ſnarling, who while they will needes have many callings, neglect their owne.* Good Sir be pacifyed: who troubleth your patience? I, whom you make your adverſary (a better friend then you deſerve) never, I aſſure you, *delivered that comparative attribution;* I diſclaime it utterly: I never made compariſons with you: you muſt ſeeke you ſome other antagoniſt. And now what is become of your angry queſtion of BETTER? it had been much better, and more for your honeſty, to have held your peace.

But we will goe on in your Pamphlet. Next followeth a ſecond Epiſtle *to the Reader*, Pag: 22, 23, Then the projecting and dividing the circles of the Ring, from pag: 24, till the end of 43. Wherein you pleaſe your ſelfe much with a portentuous invention of *your great Cylinder, for a ſtudy* (*he that will be at the charge*(*ſubaudi*) were a great foole) *of a yard diameter*: which brave conceipt doth ſo elate, or rather efferate you, that in the very next leaf you muſt needs give me a laſh: very wiſely unveyling a great ſecret about *the circle of equall parts*. After this, twenty two whole leafes being already paſt, you beginne in pag: 1, with your Pamphlet printed 1630; worthy indeed for the learnednesſe of it to be enrolled in this diſorderly band. And then in token of triumph, pag: 24, you ſet up a banner of other encomiaſticall verſes.

Let me now ſee whither we are come: and we had need looke about us: for here is a vaſt *hiatus, a huge Gulfe*. And upon an inſtant from pag: 24 we are hurryed to pag: 53: Where tenthly we have a third Epiſtle *to the Reader*, that promiſeth him wonders in *Aſtronomie, Horolography, in plaine triangles, applyed to dimenſions, Navigation, Fortification, &c.* marry this *&c.* was well put in: but it had done paſſing well two lines before. Yet you have provided well enough for all that: you have left a great *lacuna*, that what you have no skill in now, peradventure you may hereafter picke out of the labours of ſome other: and then challenge it as comming within your *intentions*: and thence ſupply your *vacuum*.

After all this he rambleth backe againe, *by way of introduction: of the examination of the graduation of the circles of the ring: which may ſerve as an inducement and furtherance to the learner, to fit and acquaint him.* What, are we no farther yet? we have fairely rowled *Syſiphus* ſtone: but to make amends, we have a few *ſcrambling* uſes in Aſtronomie, in Dyalling in plaine triangles, from pag: 56 to 67. And then the Flag of encomiaſtical verſes, of p. 24, is again gloriouſly diſplayed in pag: 68. Fy upon foolery! Fy upon vaine-glory! Fy

upon ſuch miſerable penury of matter! Now make roome: Here comes a new *projection of circles enlarged, either by a moveable and fixed circle, or by a ſingle projection with an Index at the peripheria, or center:* for here is plenty and variety. A wondrous ſecret it is, that a man may divide either one circle, or elſe foure, or ten circles, or as many as one will, into 1000 equall parts. But here our *Vavaſor* hath a worſe rub in his way. As ſpite would have it, this was firſt hit upon by one *Thomas Browne* a Ioyner: yet not one word of *Browne:* onely I am a beame in his eye. And herein lyeth a myſtery of his skill: He holdeth it no maſtery to joyne forces with a Ioyner: but by ſetting on a bold face, if with petulant inſolencies hee ſhall dare mee; hee thinketh the attempt will bee more glorious. Wherein I thanke him for putting a little difference in his eſtimation for matter of art betweene me and a Ioyner. And yet there is another matter in it too. *Browne* hath done it in a Serpentine line; and he in juſt circles: the very names of circle and Serpentine (though the things themſelves are the ſame: the ſerpentine revolution being but two true ſemicircles deſcribed on ſeverall centers) may to the ignorant (for ſuch they are that *Delamaine* muſt perſwade) ſeeme to intimate things different in nature: and ſo make good his claime againſt *Browne*. This part he cuts off ſhort in two leafes onely; reſerving all the reſt, that ought to be ſpoken thereof (which he will find harder then he conceiveth) to his large *intentions*.

But now (vvoe is me therfore) my puniſhment is at hand. All the reſt of this worthy Pamphlet, which is thirteene leafes (except the laſt page onely, which is alſo an Epiſtle *to the Reader*, the very ſame promiſing one, which was before in pag: 22, the former) is a moſt vile, unmannerly, and barbarous invective againſt me: full of untruths, full of malice, full of ſcandall, full of hypocriſy. In pag: 73 I am argued *of ſpreading unſavoury rumours*: who (God knowes) have ſcarce ſo much as thought upon him, till this ſcandalous Pamphlet came to my hands: and *of ignorance of his intentions*; whereas it partly hath, and ſhall better appeare, that I know his *intentions* well enough. Then followeth pag: 74, a fourth Epiſtle *to the Reader*, ſhort, but very quick: that *the world hath bin abuſed, as well as himſelfe, with a falſe rumour rayſed by ſome rude & ignorant tongue: by their malicious phantaſie: and that he* (good ſoule) *did not intend to take this courſe, but ſought peace, and his right by a private and friendly way: but fayling of it, his good intentions ſcorned and ſlighted, maketh the enſuing diſcourſe his plea.*

Noble Gentlemen, excuſe I pray you my moſt juſt indignation. While he was ridiculous and vaine in his opprobries, I dallyed with him

him: now this ſo deepe taxing me of want of charity, in refuſing peace ſought, and proſecuting contention and diſcord, contrary to my Chriſtian duty, pierceth to the quicke: which only ſcandal-full calumniation had it not been, I had ſcarce vouchſafed an anſwer to all the reſt. Impudent and impure mouth, for ever be thou ſtopped, that delighteſt in ſlaunder, and with lyes cutteſt like a ſharpe razor! When didſt thou ever *ſeeke peace* of me, and I refuſed it? when did I not but moſt mildly and modeſtly behave my ſelfe unto thee? returning thee good words for ill: which my Chriſtian humility thou haſt, it ſeemeth in thy pride interpreted abjectneſſe, and growne thereby more importunate and unreaſonable. What have I done? what have I ſpoken at all, with which thou canſt juſtly charge me of wrong? how many wayes haſt thou moſt intollerably provoked me; by raylings to my face, and threatning thou wouldſt *overtop* me, by letters into the countrey; and all to urge me to impatient ſpeeches, that thou mighteſt get occaſion of a ſute at law, as thy ſelfe acknowledgedſt, to have a perſonall action at the Kings bench Barre againſt me? When I was from London, thou madeſt enquiry after me and my comming up, in a diſtempered and threatning manner. When I came to London, thou ſoughteſt me out, and openly in the audience of divers witneſſes revileſt me about my book & inſtrument called *The circles of proportion* (which yet I ſet not out, nor ever ſought to make one penny benefit by) while I onely ſtood ſilent and amazed to ſee thy audaciouſneſſe and deſperate conſcience: till at laſt extreamely provoked with thy braving reproaches I onely ſayd, what ſtrange impudence is this? You know that I know what is in you, and that you have no skill in diverſe arts, which in your table you profeſſe: or if you have any, you may thanke me for it: and that you have and might have made better uſe and benefit of my friendſhip, then by theſe challenges you are ever like to get. And you anſwerd, then belike I haue all I ſhal have. And I ſaid, unleſſe you can better deſerve it. Afterwa d when I was in the countrey above a quarter of a yeare together, in deriſion of my calling you ſent ſome Porter dreſſed up lik: a wandring Miniſter, with a ſcandalous letter, full of injurious expoſtulations of wronging you in print, which I never did: of ſtealing your invention, which is as falſe: of traducing the dead: of intruding my ſelfe into your calling: and neglecting mine owne: and ſuch like peaceable ſtuffe, which letter I would not keepe, leſt by it I might hereafter be provoked againſt you: but cloſing it up againe, delivered it to the Porter, willing him to returne it backe into your hands, and bid you peruſe it with a better mind. And when afterward I ſent up to you *a Canon of ſines, tangents, and ſecants*, which I had borrowed, you asked if I had not alſo ſent ſome ſcorneſull anſwer to your letter. After this (diſpai-

ring to get any advantage out of my words) you ſhamelеſſely exclaimed upon me to my Lord Marſhall, and to my Lord of London, and to as many of the Nobility, Gentrie, and Clergie, as you thought I was known to; that ſo by depriving me of my friends and hopes, you might procure my utter undoing. Is this your *Chriſtianity?* Is this *a private and a friendly way?* is this *to ſeeke peace?* are theſe *your good intentions?* which becauſe you had not your wicked purpoſe in, you hold as *ſlighted and ſcorned:* and God grant that they may be ever ſo *ſlighted and ſcorned*, that is fruſtrated of their divelliſh intentions and deſignes, as many as have evill will againſt the innocent. Thus have you ſeene in him (honoured Gentlemen) the lively character of a querulous, clamorous, injurious, ill natured man: that like an angry curre can together bite and whine: crying out upon wrong, when hee himſelfe is the onely wrong-doer.

But reaſon is wee ſhould heare his plea: in which hee ſtill playeth his owne part, that is of ſcurrility, calumniation, out-facing, and hypocriſy. A pittifull caſe it was indeede that the world ſhould ſpie out his vanity in aſſuming to himſelfe the firſt diſcovery of the Horizontall projection and circles of proportion: *It was a malevolous diſpoſition of envious detractours: fameuſing ſome, and inſaming others: which did not a little diſturbe the quiet and peace, which formerly hee enjoyed:* when in his greedy hopes hee had ſwallowed downe the golden baite of vaine-glory, and of a large fee out of every Inſtrument the workeman ſhould ſell. *But alſo ſlacked his intentions:* or elſe there had beene yet *greater helpe for ſuch as affect Mathematicall* (he would have ſaid Manuall or Inſtrumentall) *practices: Not out of mercenary reſpect, nor interlaced* (O ſtay there: tell the very plaine truth: and ſay, Not without mercenary reſpect: interlaced) *with deluſions & bumbaſt ſtuff by way of illuſtration, if not confuſion: Had he not bin prevented by ſome others, whoſe callings might have invited them to ſpend their houres better: and which have work enough at home: whoſe ambition to bee ſomebody hath incited him forward to deliver ſome ſuppoſed new ſtuffe, or ſcrambling pieces, if not confuſed fragments of his owne, or ſome others to a publicke view, in obſcure and various phraſes: a thing ſuppoſed to be forged of ſundry heads, rather then one alone: ſeeing there is ſuch roving from the text: onely ſo farre to be truſted, as is agreeable to the text and doctrinall methode: among ſt whom to blow ſome ſmoke thereto, there was ſome groſſe one, ſeeing the matter is ſo common; for to a finer element perhaps his capacity could not aſſent, or aſcend: Yet there was ſome honeſty ſhowne not to take the crop, but the gleanings, holding it eaſyer to follow a beaten path, then to hazard a diſcovery. A blind guide, and a Parats ſpeech are not much*

different:

different: the one walkes he knowes not whither, and the other ſpeakes he knoweth not what: and ſuch are all precepts in arts, which leade, and make men ſpeake without demonſtration: which doe not onely protract the ſtudious, and fruſtrate the affectionate, but maketh an ingenious ſpirit (who is ever more rationall then practicall) to contemne ſuch circumlocutions, and laugh in private, if not in publicke, at the learned ſtile of ſome Authours making themſelves by their obſcure kind of writing ſeemingly famous, ſticke not to calumniate others to make them infamous: It is an ancient Proverbe amongſt us, Good wine needeth no buſh: But the wine muſt not be faſt lockt up then, that none can come by it, if ſo it wants both buſh and key. Excellently ſcolded: even ſo I have ſometimes at a publicke Conduit heard a Tankard-woman in her furious and rayling fit; till ſhe hath runne her ſelfe quite out of breath, and ſenſe. But *Richard Delamain*, are you ſo mad upon the fruſtrated prey of your vain-glory and lucre, that neither the ſacredneſſe of my function, nor the reverence of mine age, nor my many good deſerts, nor my innocencie of any ill demerite, nor your knowledge of ſome skill I have in thoſe ſciences, can eſcape the deriſion of your ſardonicall laughter, nor the wound of your virulent tongue? You are not in this fell paſſion to be reaſoned with. Onely I will ſoberly tell you that *William Forſter*, whom you call *a Parrat ſpeaking he cannot tell what*, is a farre more grounded Artiſt in all parts of the Mathematicks then is *R. D*: and better knoweth what belongeth to demonſtration then *R. D.* doth: as may ſoone bee tryed. And as for my *Clavis Mathematica*, at which you make your ſelfe ſo merry, though I doate not, as you doe, upon mine owne (for I ſuppoſe you will not lay claime to that too) yet I confeſſe I like it the better, becauſe it pleaſeth not your palate, to which nothing can ſavour, that is learned and Analyticall: but onely the ſuperficiall ſcumme and froth of Inſtrumentall trickes and practices. It is you ſay hard and ſhort: Did any man, I pray you, ever make a key, but of hard matter, and portable for the ſmallneſſe? and yet it openeth an entrance into the moſt magnificent ſtructures. I ſee you, and ſuch as you are, looked for an Epitome: you were deceived: It is the way of rationall Scientialliſts, not of ground-creeping Methodicks. Hee that deſireth therein any reſolution, to him I have in the Epiſtle, for the honour of my moſt illuſtrious Lord, in whoſe ſervice and family I penned it, moſt freely profered my ſelfe gentle and curteous. He that liketh it not, may let it alone. But to him that can rightly uſe that key, it will unlocke the hardeſt myſteries of thoſe Sciences, and of the writers thereof; as is not unknowne to many, who to their great contentment have beene and are verſed therein.

My calling miniſtreth to *R. D.* a diverſe and contrary matter, both of

of mirth and ſcandall. His mirth I willingly leave unto him, as not unbefitting the reſt of his good manners: for hypocriſy and prophaneneſſe may well ſymbolize together. His ſcandall, being taken at the good gifts of God, cannot alſo but bee moſt unjuſt. For it is not without impiety to be affirmed, that any part of good literature is alien and abhorrent from the calling of a Divine: but that in all ages many of the moſt eminent in the ſublimity of Theologie, have beene alſo converſant in the ſtudy of the Mathematicks; moſt profitably making them to ſerve and anciliate to their higheſt contemplations: and they that have wanted ſuch helpe, have heartily wiſhed for it, and found in themſelves the defect. And that in no other thing, after his ſacred word, Almighty God (who creating all things in number, weight, and meaſure, doth moſt exactly Geometrize) hath left more expreſſe prints of his heavenly & infallible truth, then in theſe Sciences: in which onely the mind and underſtanding of a man can find ſecure reſt and ſure footing; all other knowledges being involved with a thicke miſt of ignorance and obſcurity. Beſides, that the exerciſe of theſe Arts accuſtomed to the certainty of demonſtration, quickeneth the underſtanding, rouſing it up from a laſie and drowſie indormition and ſervile aſſent to dialecticall and conjecturall probabilities, and ſpurring it forward, and ſupplying it with meanes, unto the accurate inveſtigation of true and undeceivable principles. Now tell me R. D. are theſe ſtudies worthy of a Divine, or no? Indeed to know no more thereof then you know, that is to play with Inſtruments as a child doth with babies, or a Iuggler (though the word trouble you) with his trinkets, is unworthy of a Divine, yea of a rationall man: worthy onely of ſome rude and reaſonleſſe dulman.

But he upbraideth me for *taking libertie enough to the loſſe of time: and neglecting my calling.* I muſt confeſſe this ſcandall cutteth deepe: and hath with them, to whom I am not knowne, wrought me much prejudice and diſadvantage, in anſwering whereof I muſt crave your patience in all humble modeſty, to make a briefe recitall of the courſe of my poore laborious and painfull life.

Next after Eaton ſchoole, I was bred up in Cambridge in Kings Colledge: of which ſociety I was a member about eleven or twelve yeares: wherein how I behaved my ſelfe, going hand in hand with the reſt of my ranke in the ordinary Academicall ſtudies and exerciſes, and with what approbation, is well knowne and remembred by many: the time which over and above thoſe uſuall ſtudies I employed upon the Mathematicall ſciences, I redeemed night by night from my naturall ſleep, defrauding my body, and inuring it to watching,

cold,

cold, and labour, while moſt others tooke their reſt. Neither did I therein ſeek only my private content, but the benefit of many: and by inciting, aſſiſting, and inſtructing others, brought many into the love and ſtudy of thoſe Arts, not only in our own, but in ſome other Colledges alſo: which ſome at this time (men far better then my ſelfe in learning, degree, and preferment) will moſt lovingly acknowledge. Ever ſince my departure from the Vniverſity, which is about thirty yeares, I have lived neere to the Towne of Guildford in Surrey: where, whether *I have taken ſo much liberty to the loſſe of time, and the neglect of my calling*, the whole Countrey thereabout, both Gentry and others, to whom I am full well knowne, will quickly informe him; my houſe being not paſt three and twenty miles from London: and yet I ſo hid my ſelfe at home, that I ſeldomly travelled ſo farre as London once in a yeare. Indeed the life and mind of man cannot endure without ſome interchangeableneſſe of recreation, and pawſes from the intenſive actions of our ſeverall callings: and every man is drawne with his owne delight. My recreations have been diverſity of ſtudies: and as oft as I was toyled with the labour of my owne profeſſion, I have allayed that tediouſneſſe by walking in the pleaſant and more then Elyſian fields of the diverſe and various parts of humane learning, and not of the Mathematics onely. In all which knowledges if I have attained to no more ripeneſſe and perfection, then to be reputed, and dared out by *Richard Delamain* with ſuch contemptuous challenges, as a match ſcarce equall for him, it is ſurely a great meaneneſſe and defect of naturall gifts in mee (wherein I have juſt cauſe to be, and indeed am, humbled) and not altogether ſo much *my loſſe of time*.

About five yeares ſince, the Earle of Arundell my moſt honourable Lord in a time of his private retiring to his houſe in the countrey, then at Weſt Horſley, foure ſmall miles from me (though ſince he hath a houſe in Aldebury the pariſh where I live) hearing of me (by what meanes I know not) was pleaſed to ſend for me: and afterward at London to appoint mee a Chamber in his owne houſe: where, at ſuch times, and in ſuch manner as it ſeemed him good to imploy me, and when I might not inconveniently be ſpared from my charge, I have been moſt ready to preſent my ſelfe in all humble and affectionate ſervice: I hope alſo without the offence of God, the tranſgreſſion of the good Lawes of this Land, neglect of my calling, or the deſerved ſcandall of any good man. And *R. D.* too, (if hee had ſo much grace or wit) may taxe himſelfe of malapert ſawſineſſe to call in queſtion the priviledges and wills of Noblemen, the diſpenſations of the Lawes, and the conſciences of others, by ſuch uncharitable and ſcandalous cenſures. But hee and his like muſt be ſuf-

 fered

fered to preſcribe lawes for others, and not ſo much as keepe good manners themſelves.

And although I am no *mercenary man*, nor make profeſſion to teach any one in theſe arts for gaine and recompence, but as I ſerve at the Altar, ſo I live onely of the Altar: yet in thoſe interims that I am at London in my Lords ſervice, I have been ſtill much frequented both by Natives and Strangers, for my reſolution and inſtruction in many difficult poynts of Art; and have moſt freely and lovingly imparted my ſelfe and my skill, ſuch as I had, to their contentments, and much honourable acknowledgement of their obligation to my Lord for bringing mee to London, hath beene teſtifyed by many. Of which my liberallity and unwearyed readineſſe to doe good to all, ſcarce any one can give more ample teſtimony then R. D. himſelfe can: would he be but pleaſed to allay the flame of this his hot and eager contention, blowne up onely with the full bellowes of intended glory and gaine; and to ſpeake the truth. Yea neither is hee ſo unkind: but *ſome furtherance from mee in triviall matters hee doth and ſhall acknowledge freely*. This were an honeſt profeſſion if it were with gratitude, and for love of the truth, and not to aſſert a greater untruth: See his cunning: thus he argues: *I had not in the Horizontall Inſtrument the leaſt touch of furtherance from him or any man breathing, either by tranſcript or verball direction*: for if I had, it may be preſumed, I ſhould as ingenuously have confeſſed it, as I doe freely acknowledge his furtherance in ſome other things. A fine piece of Sophiſtry that *Ariſtotle* never taught; by confeſſing a truth to averre a lye. And marke how *cautelous and ſubterfugious* (though he jeſt at the words) his acknowledgement is: *other triviall matters*. What doe you here acknowledge, when you reſerve power to deny every particular thing? Well, wee will take what you pleaſe to beſtow, they were *triviall matters*. Such a learned Authour as you are, to be furthered in *triviall matters*? If you need ſuch *furtherance in triviall*, we ſhall ſuſpect you in greater. Becauſe you ſcorne to mention ſuch *triviall* things, I will helpe you out with them: they were the firſt elements of Aſtronomie concerning the ſecond motions of the fixed ſtarres, and of the Sunne and Moone: they were the firſt elements of Conics, to delineate thoſe ſections: they were the firſt elements of Optics, Catoptrics, and Dioptrics: of all which you knew nothing at all. And diverſe things alſo which you profeſſe, whereof you knew very little. I recite not theſe things for exprobration: but that you may a little remember your ſelfe.

I have, I hope, even now cleared *my calling*, and claime that I may make to theſe arts of Mathematics, ſo far-forth as I uſe them

I may:

I may therefore with better manners aske you, how you obtayned *that calling and profeſſion:* for you challenge both names to your ſelfe. What Vniverſity, what degree, what court of faculties, what other lawfull way, conferred it upon you? I beleeve you can anſwer me never a word: but will be horribly to ſeeke in your *plea*. Well, I will ſtand your friend once more, and helpe you out, and derive you a faire title to the inheritance of a *vulgar Teacher*. When you had learned to reade, you went to the Writing ſchoole: and can indeed if you liſt write a faire hand. Then you learned over your accidence: Afterwards, I heard you ſay, you went into France (it may be to the Iſle Iernſey) where your name got the French garbe: but little or nothing of the tongue brought you home with you. Next you tooke the degree of a Inſtices Clearke, or a Doctours of Phyſick, or both: to make Warrants or Mittimus, or it may be Recipe's, provided they were not in Latine, or in French. From thence you were advanced to keepe a Writing ſchoole in Drury lane: and ſo had opportunity to heare the Lectures at Greſham Colledge: and to have the benefit of conference with learned men. When you now thought you could cant in the Inſtrumentary idiome, you requeſted *Iohn Thomſon* the maker of Mathematic Inſtruments in Hoſier lane, to helpe you to ſome Schollers. And is not this a faire pretence to the Mathematics: which you doubt not to call *Our noble profeſſion, and our profeſſion of ſo noble a Science?*

But leſt I may ſeem to make good that crime of *Detraction* wherewith he doth charge me, by detracting from him both French and Latine, contray to the faſhion of his name, and the many ſhreds and thrummes of Latine he doth ſo artificially weave into the web of his Pamphlet, I will without any *ſlander* tell you a true ſtory. Betweene foure and five yeares agoe, a young Dutch Gentleman whoſe name was *Dunbeſt* comming into this Land, ſojourned in a friends houſe of mine in London: and becauſe the Gentleman addicted himſelfe to the warres, hee was deſirous to have the helpe of ſome learned Teacher of the Mathematics. My friend thinking *Richard Delamain* to be ſuch an one, ſent for him; to whom the Gentleman ſpake (I cannot ſay ſignifyed) his deſire in Latine: but our learned *Profeſſor* ſtared him in the face as if he wondred, but anſwered him not: which the Gentleman perceiving ſpake in Frenchibut that was more ſtrange: the Gentleman therfore making uſe of ſuch little Engliſh as he had gotten, asked him, cannot you ſpeak Latine? *No.* Can you not ſpeake French? *No.* How ſhall I then that underſtand not Engliſh learne of you? And ſo our grand Maſter went away as wiſe as he came without his Scholler. which great misfortune of that poore young man to loſe ſuch learned *fundamentall Mathemati-*

call Doctrine may be a faire warning for all Gentlemen ſtrangers to get them an Engliſh tongue in their heads, and that quickly: or elſe they are not like to have *their ſight holpen* by this our great oculiſt and *unvayler of the ſubject Richard Delamain.* But here by the way *ſome malevolous Detractor may ſpightfully* collect, that if our *Profeſſors* Latine, and French, and Greeke be but meere contrefaict, which yet he doth ſo ventilate for his glory: his Mathematics may well be ſuſpected to be of the ſame ſtuffe.

God knowes how unwillingly and with how grieved a mind I write theſe things, or ſo much as put pen to paper againſt him: But moſt indignous and inſufferable are the abuſes offered by him to me, his ſcandalls, calumniations, bravings, and outfacings, and all mixed with more then Thraſonicall arrogancy, throughout his whole Pamphlet: which that hee may bee ſure to ſcatter every where, he ſendeth up and downe to his acquaintance by halfe doſens: and therewithall a letter, wherein he both requeſteth to have them diſperſed, and nameth to whom: and alſo bitterly inveigheth againſt me, and threatneth me. ſome of which letters have bin ſhewed to me: and it may be I ſhall prevaile to have them produced. Beſides in his daily talke to every man he baſely traduceth me, and gloryeth in reading unto them his Pamphlet, and his letter which he ſent me into the Country, marveilouſly pleaſing himſelfe at the ſporte he maketh with his ſcoffes and jeſts, acting them with his hands and the geſture of his body, and ſaying *here I come over him finely, here I give him a laſh, here I ſcourge him,* with other ſuch like contemptuous ſpeeches. And alſo ſendeth to me ſometimes threatning, ſometimes ſcornefull meſſages: challenging, and even daring me to make him an anſwer. What ſhould I, what can I doe in this caſe? If I let him alone in all theſe his deſpightfull and inhumane injuries; all men may ſcorne me, and the very boyes in the ſtreet point at me; and he (as hitherto he hath done) by my patience and meekeneſſe grow into a higher degree of pride and inſolencie, and be more obſtirmed. I ſpeake unfeignedly, that in my heart I pitty him: and wiſh him not the leaſt hurt: for he needeth it not: but this he needeth, to repent, and be humbled, that he may know himſelfe, and his friends. I could have written much more, and more ſharpely: but leſſe then I have done, and with greater mildneſſe (conſidering the haynouſneſſe of his injuries, not only in print reviling and diſgracing me publikely, but alſo by ſecret ſlanders and malicious clamourings labouring utterly to diſcredite and undoe me) I could not write.

The Inſtruments I doe not value or weigh one ſingle penny. If I had

I had been ambitious of praiſe, or had thought them (or better then they) worthy, at which to have taken my riſe, out of my ſecure and quiet obſcuritie, to mount up into glory, and the knowledge of men: I could have done it many yeares before this pretender knew any thing at all in theſe faculties. And when at *William Forſters* requeſt I was contented to give way that he might publiſh them, I had not the leaſt thought to be ſeene or acknowleded by them: but only to gratify and doe ſome good to *Elias Allen*, whom he very ſpitefully, yet more fooliſhly (contrary to the generall repute had of him in this and other lands) termeth *an unexpert Workeman*. Now judge, I beſeech you, had it not beene extreame ſimpleneſſe in me, to ſtand by, and hold the candle; while a vaineglorious braggard, who had by mine, and *Elias Allens* meanes gotten the overture of thoſe Inſtruments, ſhould ſo perk up himſelfe in ſtolne feathers, and audacicuſly outface me in mine owne: and make *Elias Allen* his farmer for my free gift, not to worke, but at his devotion, and for his profit? Might not I then juſtly have beene laughed at, and ſtiled the Bawde and Pandar of the vaine-glory, and ſhameſull lucre of *Delamain*?

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Of the Horizontall Inſtrument.

LOng agoe, when I was a young ſtudent of the Mathematicall Sciences, I tryed many wayes and devices to fit my ſelfe with ſome good Diall or Inſtrument portable for my pocket, to finde the houre, and try other concluſions by. and accordingly framed for that my purpoſe both Quadrants, & Rings, and Cylinders, and many other compoſures. Yet not to my full content and ſatisfaction: for

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Of the Horizontall Inſtrument.

LOng agoe, when I was a young ſtudent of the Mathematicall Sciences, I tryed many wayes and devices to fit my ſelfe with ſome good Diall or Inſtrument portable for my pocket, to finde the houre, and try other concluſions by. and accordingly framed for that my purpoſe both Quadrants, & Rings, and Cylinders, and many other compoſures. Yet not to my full content and ſatisfaction: for

either they performed but little, or els were patched up with a diverſity of lines by an unnaturall and forced contexture. At laſt I conſidering that all manner of queſtions concerning the firſt motions were performed moſt properly by the Globe it ſelfe rectifyed to the preſent elevation, by the helpe of a moveable Azumith: I projected the Globe upon the plaine of the Horizon, and applyed to it at the center, which was therein the Zenith, an Index with projected degrees, for the moveable Azumith. in which projection I firſt found what I had before with much ſtudie and paines in vaine ſought for. And becauſe I ſeldomely came to London, where I might have the helpe of large Compaſſes, and other Inſtruments, for drawing the arches of very big circles: I was forced to betake my ſelfe to ſuch ſhift, as Art would afford me: and invented many Theoremes, problemes, and practices (ſuch as no man before, that ever I could find, had delivered) for the finding out of the interſections, and all and every points of all thoſe circles, by which I might draw the ſame, and divide them being drawne. Which rules I have yet in my paper booke, carrying their antiquity in their very ſhew: and are acknowledged by this challenger to have beene ſeene by him. And though I invented them being young, yet they will paſſe the skill of his glorioſity, but even fitly to apply them to uſe, much more to *demonſtrate* them.

About thirty yeares ſince I preſented one of them drawne with mine owne hand to the truely reverend Prelate Doctour *Byſſon* Biſhop of Wincheſter, by whom I was made *presbyter*.

About five and twenty yeares agoe I beſtowed one upon a noble Ladie, the wife of a worthy and learned Knight, then abiding neere the place where I live, but ſince dwelling in Worceſterſhire; which Lady with ingenioufneſſe and ſolertie more then fœminine tooke delight in the ſpeculation and uſe of the Globe. And for her I writ many notes upon my Inſtrument, the very ſame almoſt word for word, which many yeares after I ſent in a letter to *Elias Allen*; and are they which *Delamain* acknowledgeth to have *ſeene*, but *ſlighteth*. I remember I did upon that Inſtrument tricke out in colours and mettall, the coate armes of both thoſe families joyned in pale, the draught of which armes I yet have together with thoſe rules. And I doubt not but that noble Lady doth as yet keepe that little Inſtrument; and will be pleaſed for the vindication of my credite to produce the ſame.

In the Spring 1618 I being at London went to ſee my honoured friend Maſter *Henry Brigge* at Greſham Colledge: who then brought me

me acquainted with Maſter *Gunter* lately choſen Aſtronomie reader there, and was at that time in Doctour *Brooks* his chamber. With whom falling into ſpeech about his quadrant, I ſhewed him my Horizontall Inſtrument: He viewed it very heedfully: and queſtioned about the projecture and uſe thereof, often ſaying theſe words, it is a very good one. And not long after he delivered to Maſter *Briggs* to be ſent to me mine owne Inſtrument printed off from one cut in braſſe: which afterwards I underſtood he preſented to the right Honourable the Earle of Bridgewater, and in his booke of the Sector printed ſixe yeares after, among other projections ſetteth down this: herein ingenuous that he did not challenge it to himſelfe (as our challenger doth) but not ingenuouſly enough acknowledging from whom he had it. But ſuch is the providence of God, I kept that very letter of Maſter *Briggs* wherein he ſent me that print from Maſter *Gunter*, dated from Greſham Colledge 2 *Iun.* 1618: and the poſtſcript 4 *Iune*: and which came to my hands *Iune* 10. In which letter are theſe wordes *Maſter Gunter doth here ſend you the print of a Horizontall Dial of his drawing after your Inſtrument.* This very letter hath beene left by me in the hands of *Elias Allen* above theſe two yeares to be ſeene of any one that will require it. Yea and our challenger himſelfe in his Epiſtle to the Reader before his booke of the Horizontall quadrant doth acknowledge the ſight of this letter, and ſetteth downe the very words. Which maketh me wonder at the ſtupidity of his audaciouſneſſe, ſo without all ſhame and ſenſe contradicting himſelfe. Vnleſſe he thinke to have this evaſion, that I deviſed the projection, but knew not the uſe of it when I had done. I preethee *R. D.* why did I ſhew it to Maſter *Gunter* then? was it only for the pictures ſake? And what did he like it for? becauſe it was ſo fairely lineated? Or was it not for the excellent and copious uſe it hath above any other Inſtrument of that nature? But heare his *plea*, or rather his play and jugling with God and man, and his owne conſcience: *The extendure of Gods hand in his donations is manifold, and where his ſpirit pleaſeth to breath there is a doore opened: they poſſeſſe the world with a contrary opinion, thereby wronging God in his diſpenſation, and men in his reputation.* Gentlemen, doth not your haire ſtand an end with horrour at ſuch prophane hypocriſy? for ſhame repent. but why doe I call for ſhame where is none?

About two yeares after I had ſhewed that my Inſtrument to Maſter *Gunter*, I beſtowed the very ſame individuall one upon a young Gentleman, now a Baron, my very honourable and moſt intire friend, a man full of vertue, full of learning, full of all goodneſſe, and true nobility, whoſe only defect and fault is an unquenchable thirſt after knowledge and good literature; who hath yet the very

very ſame in his cuſtody: and is at this preſent in London: whoſe honourable word and teſtimony will confirme that he himſelfe ſo many yeares agoe knew the uſes of that Inſtrument: and yet our chalenger never *unvayled* it to him: nor dareth preſcribe for ſo long time.

In Michaelmas Terme 1627 I came to London, and *Elias Allen* having beene ſworne his Majeſties ſervant, had a purpoſe to preſent his Majeſty with ſome New-yeares gift: and requeſted me to deviſe ſome pretty Inſtrument for him. I anſwered, that I have heard his Majeſty delighted much in the great concave Dyall at White-hall: and what fitter Inſtrument could hee have then my Horizontall, which was the very ſame repreſented in flat? and that I would upon the backeſide ſet the theorics of the Sun and Moone. And ſo by helpe of both ſides Eclipſes might be calculated with great facility. He liked it well. The Horizontall ſide was begunne by my direction. I was not long at home, but Maſter *Allen* being at a ſtand in his worke, ſent to me for helpe. I writ him a large letter two ſheets of paper long: wherein I taught him the uſes of the Inſtrument eſpecially the Horizontall: and afteward the fabric or delineation of it: and how to find the ſemidiameters and centers of the ſeverall circles both great and leſſer, and the way to divide them. Which letter Maſter *Allen* yet keepeth: and is the ſame I ſpake of before: and which *Delamain* confeſſeth he *ſaw*.

Obſerve here I pray you, *the ſubject* even by his owne confeſſion *was unvayled* before he medled with it. And I would to God Maſter *Allen* had in good time finiſhed up that Inſtrument: I wiſh it for the challengers ſake: it might have ſaved him from a great deale of ſinne and ſhame. But hereby we may diſcover his worthy *intentions*, whereof he braggeth ſo much: Hee ſeeing Maſter *Allen* to neglect it, and my ſelfe not to make any great account of it, tooke it up as a wayft or ſtray: and had a purpoſe long agoe to have *famouſed* himſelfe thereby: firſt calling it the *Grammelogia*: And then had hee been pittifully to ſeek of a new name for the Circles of Proportion, an Inſtrument not yet in rerum natura with him: for now his Greek Nomenclator, and oracle the Schoole-maſter of Saint Clements was defected by death. Yet the name *Grammelogia* would ſerve as well for the other Inſtrument, as ſoone as he had heard of it, although by a ſpightfull accident he (being not yet aſcended to the height of a profligated ſhamelesneſſe) was hindred in the production of that his firſt plagiarious birth.

For ſome good tract of time after this, when I was now in my Lords

Lords ſervice, and *Delamain* frequented my chamber: One day after he was gone downe: another man came up and told me, that *Delamain* was in Maſter *Allens* ſhop ſhowing unto diverſe a little Inſtrument in braſſe of a triangular or rather harpe-like forme, with which he could performe all the queſtions of the Globe for any part of the world, and make Dialls, and deſcribe Countries, and carry Mines under the earth as farre as betweene Temple barre and Weſtminſter, and ſuch like wonders, which I knew impoſſible for any ſuch Inſtrument to performe. I ſaid ſurely he miſtooke: for but now he went hence: and had neither then, nor at any other time, ever ſpoken of any ſuch matter: which I was ſure he would have done, had he any ſuch thing in his mind: But he ſtill affirming it: I had a great deſire my ſelf to be a witneſſe of that wonder. I came to *Elias Allens* ſhop; but he was gone. I told *Elias Allen* what I had heard: and ſaid I would goe to his houſe, and ſee it. I came to his houſe pretending ſome other occaſion. He ſhewed me a great quadrant of Gemma Friſius he had begunne: and after that a quarter of the Analemma: which I viewing told him that the Meridians were falſely drawne. Indeed ſaid he I cannot make them anſwer to any Center. Whereat I ſmiling ſayd, it is no marvell, for they are not arches of Circles: and ſhewed him the reaſon why they could not be. What are they then ſaid he? They are Ellipſes ſaid I. Ellipſes ſaid he, what is that? I told him: and diſcourſed of the kinds of Conic ſections, the firſt newes that ever he had heard of any ſuch thing. Well, at laſt I asked him for the ſtrange Inſtrument he had ſhewed: and would not be anſwered but he muſt needs ſhew it me: which with much tergiverſation he did. Tuſh ſaid I, this is nothing but halfe my Horizontall. which he alſo acknowledging: I asked who drew it? my ſelfe ſaid he. Is it poſſible ſaid I that you that cannot make the Analemma, ſhould draw this projection? Doe you know the uſe of it? Yes ſaid he: I have written ſome notes of the uſes of it: and ſhewed me ſome papers: which I looking upon ſaw the very notes I had declared in my letter to Maſter *Allen*: but here and there the words diſguiſed after his owne apprehenſion. I went homeward: and ſeeing Maſter *Allen* in his ſhop, ſaid to him, I pray anſwer me a queſtion, but anſwer me truely. he perceiving what I meant to aske, prevented me with theſe words, indeed I did: he had the letter of me a whole fortnight, almoſt as ſoone as you ſent it: and I beleeve he writ it out: for the ſummer following, unknowne to me, he got my ſervant to make it for him: for which I was angry. The reſt of this buſineſſe let Maſter *Allen* himſelfe tell you.

Well: this might have beene all ſpared, you will ſay: the ſight

of that letter and of thoſe uſes is confeſſed: but *they were ordinary, meane, and triviall: and be ſlighted them.* That my very letter is yet extant at Maſter *Allens* making appearance to anſwer to the diſgracefull taxations of *Richard Delamain.* In which letter dated *December* 3. 1627. you ſhall find theſe uſes following.

" 1 To find the declination of the Sunne every day.
" 2 To find the courſe of the Sun; or the parallel which the Sun
" runneth, or deſcribeth every day.
" 3 To find the riſing of the Sunne, and his ſetting; and the di-
" urnall arch or length of the day, or of the night.
" 4 To find the diſtance of the Sunnes riſing and ſetting from the
" Eaſt and Weſt points, Northward in ſummer or Southward in
" winter, called the Amplitude Ortive.
" 5 To find the true place of the Sunne on the Inſtrument at any
" time of the day.
" 6 To find the houre of the day.
" 7 To find the Azumith or verticall circle in which the Sunne
" is: or the Horizontall diſtance of the Sunne from the Meridian.
" 8 Againe the Azumith of the Sun being given, to find the alti-
" tude of the Sunne, and the houre.
" 9 To find at what houre the Sunne commeth to be full Eaſt or
" Weſt every day in ſummer
" 10 To find the height of the Sunne at high noone every day, and
" likewiſe at every houre. Whereby is made Maſter *Gunters* Qua-
" drant, and all other Quadrants of that ſort, deſcribed by *Gemma*
" *Friſius, Munſter, Clavius,* and others: alſo all manner of Rings,
" Cylinders, and innumerable other Topicall Inſtruments, for the
" finding out of the houre, and other like concluſions. And like-
" wiſe the reaſon of finding out the houre of the day by a mans
" ſhadow: or by the ſhadow of any gnomon ſet up perpendicular to
" the Horizon, or elſe parallel to it.
" 11 To find out the Meridian line, and the points of the Com-
" paſſe without a needle: yea more exactly then with a needle.
" 12 To find the declination of any wall.
" 13 To find at what houre the Sunne commeth unto any wall, or
" window every day in the yeare: as alſo when it leaveth it.
" 14 To find how many, and what houre lines are to bee drawne
" in every plaine Diall.
" 15 To find how low the Sun is under the Horizon at any houre
" of the night: and in what point of the Inſtrument the true place
" of the Sunne then is.
" 16 To find in which of the twelve houſes the Sunne is at any
" time of the day or night.

" 17 To

« 17 To find the length of the crepuſculum or twy-light every day.
« 18 To find out the houre of the night.
« 19 To find the ſigne and degree in which the Sunne is every day.
« 20 To find the declination of thē Sunne every day.
« But I could beſide theſe adde diverſe other operations to be per-
« formed by the Inſtrument as now it is: and many others with
« ſome additions to the Inſtrument: as namely the degree of the
« Æquinoctiall in the Meridian at any time: and the degree of the
« Æquinoctiall in the Horizon Eaſt and Weſt: and the degree of
« the Zodiac in the Meridian called Cor cœli: and the aſcendent
« degree thereof called the Horoſcope: and concerning the twelve
« houſes of the heavens for the erecting of a figure: and concer-
« ning the ninetyeth degree of the Ecliptic above the Horizon, and
« the altitude of it. and I know not what elſe, or rather almoſt any
« thing elſe.

Theſe are *the ordinary, meane, and triviall uſes*, which I delivered, and are to be ſeene in my letter. And hath *Delamain unvayled* any I doe not ſay more (for he runneth diviſion) but other uſes then I have done? Yes marry hath he. for in his booke of his *Horizontall quadrant*, from pag: 44 to 51, you ſhall find theſe uſes.

Eighth-ly, to find the inequality of time in equall moneths, or equall number of dayes.

Ninth-ly, to find the degree of the Æquator in the Horizon by ſuppoſing the degree of the Ecliptic in the Horizon.

Tenth-ly, to find the degree of the Ecliptic in the Horizon by ſuppoſing the degree of the Æquator in the Horizon.

Eleventh-ly. But if the degree of the Ecliptic in the Horizon were required by knowing the degree of the Ecliptic in the Meridian.

Twelf-ly, to find the Horoſcope, or the degree aſcendent, or deſcendent, and the Nonageſime degree at any houre.

Thirteenth-ly, to find what angle the Ecliptic maketh with the Horizon, or the altitude of the Nonageſime degree of the Ecliptic above the Horizon: and what Azumith it is in at any houre.

O Sir (may *Richard Delamain* ſay) now I have *overtopped you*: in theſe things you cannot deny, but that *I have unvayled the ſubject to helpe your ſight*. Not ſo neither: for every worke is aſcribed to him that firſt found it out. Nor is the Authour therfore to be accounted *ignorant, or to want ſight*, though ſome other after him ſhall make ſome addition or acceſſe thereto: ſeeing it is an eaſy matter to adde to an invention once diſcovered. But yet let us ſee what learned and rare uſes thoſe are, which you have *unvayled*.

The *eighth* is utterly alien from this Inſtrument: and requireth neceſſarily the knowledge of the true, and proper motion of the Sunne, which this Inſtrument giveth not at all: and of the exact right aſcenſion, which this Inſtrument giveth but at large: and ſo is this *uſe* of no uſe, but a vaine flouriſh.

The *ninth* is nothing elſe but to find out the Sunnes oblique aſcention.

The *tenth*, *eleventh*, *twelfth*, and *thirteenth* (which indeed were excellent uſes, if he could ſhew them) are utterly falſe. In all which you have *unvayled* nothing but your owne want of skill, and moſt groſſe ignorance of the very ground of this projection. And now have you not very fairely *helpen my ſight*, and the ſight of others, to ſee your raſhneſſe and lacke of art? which all your facing (though your face, if it be poſſible, were harder then it is) will never bee able to make good.

Yet for all this (and now I challenge you) let us ſee the performance of theſe queſtions upon the Horizontall Inſtrument, with what reaſonable addition you can, which ſhall not quite alter the nature of it: and I will freely acknowledge you to be a man of art: and not at all impute unto you any *plagium*, or Mounte-banke tricks.

But ſeeing you have already *unvayled* your want of wit, I will take a little paines for you to *unvayle* your want of honeſty; *to helpe the ſight of* theſe Gentlemen our judges, to ſee what truſt they may repoſe in ſuch an Inſtrument-monger and player of leger-de-la-main, as you are. While hee was printing his tractate of the Horizontall quadrant, although he could not but know that it was injurious to me in reſpect of my free gift to Maſter *Allen*, and of *William Forſter*, whoſe tranſlation of my rules was then about to come forth: yet ſuch was my good nature, and his ſhameleſſeneſſe, that every day, as any ſheet was printed, hee ſent, or brought the ſame to mee at my chamber in Arundell houſe to peruſe. which I lovingly and ingenuouſly did, and gave him my judgement of it. When we were come to the ſaid pag: 44, to 51, I gently ſhewed him the falſity of thoſe propoſitions. And he ſaid, cannot they be wrought then? No, replyed I, not by the Inſtrument as now it is, without ſome addition. I can worke them; but you cannot. he asked, why cannot I as well? I anſwered, becauſe you are ignorant of the ground of this Inſtrument and projection. What ſhall I doe then? ſaid hee. you muſt, ſaid I, be content to loſe that ſheet, and new print it: After a little pawſe, he was not aſhamed to reſolve with theſe very words, *though your*

your ſight be ſo ſharpe, that you can note theſe faults, yet many hundreds that ſhall ſee the booke will never be able to ſpye them. and withall told me that he had penned that booke in a fortnight with great haſt: I ſaid I did eaſily beleeve as much: for *Canis feſtinans cæcos parit catulos.* This was at that time our communication, and his gallant reſolution. And if this be not jugling, never did any Hocus-pocus jugle. That unleſſe a man were given over to ſhame and ſhameleſneſſe: he would never ſo ſhamefully abuſe his learners, and ſo ſhameleſſely hazard his (I cannot ſay good) name, and reputation. Yet ſticketh he not moſt vainely (that I may ſay no worſe) to conclude his ſaid booke with this braving flouriſh, *But if any man deſire to ſay more upon this Horizontall quadrant, then I have done, I have made way for him, and unvayled the ſubject to helpe his ſight.*

But he ſaith the projection was none of mine: for *Munſter* hath it and *Blagrave*, and ſome others, this latter writ ſome yeares ſince I beganne to uſe this Inſtrument: and that in *Munſter* is no projection, but a reſemblance of a concave Diall: which likeneſſe can no more argue this Inſtrument, then *Delamains* blacke clothes can prove him to be a ſcholler. And it were a wonder, that ſeeing the writers of theſe Arts doe imagine their Diagrammes upon the plaines of ſeverall Circles, as occaſion requireth: if none ſhould be found that have made their delineations upon the plaine of the Horizon. But of ſuch as ever have uſed the ſame for an Inſtrument, before me, he neither can, nor hath ſhewed any.

Of the Circles of proportion.

FOr theſe I muſt freely confeſſe, I have not ſo good a claime againſt all men, as for my Horizontall Inſtrument: though againſt *Richard Delamain* I have. The honour of the invention, next to the Lord of Merchiſton, and our Maſter *Briggs*, belongeth (if I have not been wrongly informed) to Maſter *Gunter*, who expoſed their numbers upon a ſtreight line. which being once done, was there any ſuch maſterie to bring the ſame line about into a circle? And what doth this new Inſtrument (call it the *Circles of proportion*, or call it the *Ring*, or what other name you liſt) ought elſe, but onely bowe and inflect Maſter *Gunters line or Ruler.*

The manner how I fell upon it, was thus. I have in my ſtudie and practice of the Mathematics been not a little converſant in calculation. And that I might both facilitate the labour, and try the worke: I invented many ſolerties and compendiations in logiſtica, for the

one: and framed diverſe kinds of Inſtruments and mechanicall practices, for the other: that when I ſhould find the performance in both wayes not to diſagree, I might be aſſured of my juſt dilligence in numerary computation. Among other Inſtruments I much liked the ſame line or Ruler: onely this defect I found that it required many times too great a paire of Compaſſes, which would bee hard to open, apt to ſlip, and troubleſome for uſe. I therefore firſt deviſed to have another Ruler with the former: and ſo by ſetting and applying one to the other, I did not onely take away the uſe of Compaſſes, but alſo made the worke much more eaſy and expedite: when I ſhould not at all need the motion of my hand, but onely the glancing of my ſight: and with one poſition of the Rulers, and view of mine eye, ſee not one onely, but the manifold proportions incident unto the queſtion intended. But yet this facility alſo wanted not ſome difficulty, eſpecially in the line of tangents, when one arch was in the former mediety of the quadrant, and the other in the latter: for in this caſe it was needfull that either one Ruler muſt bee as long againe as the other; or elſe that I muſt uſe an inverſion of the Ruler, and regreſſion. By this conſideration I firſt of all ſaw that if thoſe lines upon both Rulers were inflected into two circles, that of the tangents being in both doubled, and that thoſe two Circles ſhould move one upon another; they with a ſmall thread in the center to direct the ſight, would bee ſufficient with incredible and wonderfull facility to worke all queſtions of Trigonometry both right-lined and Sphæricall. And according to this my ſpeculation, above twelve yeares agoe, I with mine owne hand made me two ſuch Circles, which I have uſed ever ſince, as my occaſions required.

In the long vacation 1630 I ſhewed both the Rulers, and the Circle to *William Forſter* (ſomewhat more præmaturely then for the deſire I had to leade him on in the right way of Art, I intended) at my Parſonage houſe, as in his Epiſtle before his Tranſlation hee doth himſelfe teſtify. To whom, exhorting me to publiſh them, I ſaid I would not appeare to the world in ſuch toyes: but if hee would take the paines to tranſlate ſome rules I had written into Engliſh, we would beſtow upon *Elias Allen* (if he ſhall thinke they may bee beneficiall to him) both thoſe Circles of proportion, and alſo another Inſtrument, conſiſting of two halfe circles moſt plainely and eaſily giving the Proſthaphæreſes of the Plannets according to the Theory of *Copernicus* (which I have had fairely drawne with mine owne hand above theſe twenty yeares) which might be ſet upon the other ſide of the plate: and would together make up the moſt compleat Inſtrument for all Aſtronomie, that ever yet to my knowledge came forth. And of this intire Inſtrument at my comming up to London

don in Michaelmas Terme following, to attend my ſervice, I did accordingly make a moſt free donation to *Elias Allen* by the ingagement of my promiſe. And had alſo performed it long agoe (I doubt not but to your good contentment) had not this Trifler ſo unſeaſonably blurted out his ſcrimble-ſcramble of *Grammelogia*, like an unlicked Bear-whelp. Wherein under ambiguous words, and large unlimited *intentions, and the generall names of Circles, Rings, and Grammelogia or declaration of lines*, hee endeavoureth very honeſtly to hooke in within his privilege, and to faſten upon as his owne, whatſoever invention any other Artiſt ſhall in a round or circular forme hereafter produce.

Will you be pleaſed to have an inſtance of this? Shortly after my gift to *Elias Allen*, I chanced to meet with *Richard Delamain* in the ſtreet (it was at Allhontide) and as we walked together I told him what an Inſtrument I had given to Maſter *Allen*, both of the Logarithmes projected into circles, which being leſſe then one foot diameter would performe as much as one of Maſter *Gunters* Rulers of ſixe feet long: and alſo of the Proſtaphæreſes of the Planners and ſecond motions. *Such an invention have I* ſaid he: for now his *intentions* (that is his ambition) beganne to worke: but how wiſely you ſhall ſee. He not conſidering the proportion of the circumference to the Diameter, which is more then triple, dreamed that I underſtood *a Circle of ſixe feet Diameter, by it to worke the Proſthaphæreſes*: as you may ſee in the very end of his tractate of *Grammelogia*. which ſo monſtrous conceipt never entred into my mind. but this may ſerve as a faire *intention* to lay claime to my Proſthaphæreticall Inſtrument, if ever it ſhall come forth: whereof hee knoweth no more then the cap upon his head.

But he ſaith, *Then after my comming home I ſent him a ſight of my projection drawne in paſt-board.* See how notoriouſly he jugleth without an Inſtrument. *Then after*: how long after? *a ſight of my projection*: of how much? More then ſeven weekes after on December 23, he ſent to mee the line of numbers onely ſet upon a circle: which I marvell he ſhould be ſo ſimple to boaſt of, *ſeeing Joyners, and Carpenters, and other Mechanics about this towne, and elſewhere, yea and ſchoole-boyes, in imitating Maſter Gunters Ruler* incurvated only into a circle, might have, and ſome have drawne, to more good purpoſe then ever yet *Delamain* did. and ſo much onely he preſented to his Majeſty: but as for Sine or tangent of his, there was not the leaſt ſhew of any. Neither could he give to Maſter *Allen* any direction for the compoſure of the circles of his Ring, or for the dividing of them: as upon his oath Maſter *Allen* will teſtify how hee

miſled

miſled him, and made him labour in vain above three weeks together, untill Maſter *Allen* himſelfe found out his ignorance and miſtaking. which is more cleare then is poſſible with any impudence to be outfaced. Yea and the concluſion of his tractate of *Grammelogia* pag: 22 doth plainly enough intimate as much: ſaying, *If there be compoſed three rings, &c. So if you move the ſine of 90 degrees unto the Tropicall point, &c. Againe in the Circle of the tangents if you bring, &c.* Where you ſhall find his deepe *intentions* ſet downe in words then which Sphynx it ſelfe never had more implexed: and teacheth nothing, but mocketh his Reader, as I have ſomtimes ſeen a child crying for a wild bird deluded, with bidding him get the bird, and lay ſalt upon the tayle, and ſo he ſhould catch it.

It will not be impertinent, but peradventure much to the purpoſe, that you may rightly know our Challenger, to let you underſtand how he hath dealt with the Ioyner *Thomas Browne* of whom I ſpake before. *Richard Delamain* hearing that *Brown* with his *Serpentine* had *another line* by which he could worke to minutes in the 90 degree of fines: uſed a device to get *Browne* to come and bring his inſtrument to ſome place where he himſelfe ſhould alſo be: there he inſinuateth with *Browne* pittifully complaining of the wrong Maſter *Oughtred* had done him, and to make the matter good readeth to him in his Pamphlet, glorying much how he had laſhed me therin: and then gave the booke to *Browne*: who in thankfulneſſe could not but gratify *Delamain* with his *Lines* alſo: and teach him the uſe of them, but eſpecially of the *great Line*: with this caution on both ſides, that one ſhould not meddle with the others invention. Two dayes after *Delamain* ſent a Porter to *Browne* for the booke he had given him, becauſe he had found ſome things to be altered therin, and that he would for it give him a better and more perfect book. *Brown* refuſing to ſend it by the Porter, the next morning *Delamain* vouchſafed himſelfe to come to his houſe neare Algate: and asked for the booke, (*Browne* ſuppoſing he would then have corrected it) but as ſoone as he had got it in his hands he rent out all the middle part with the two great Schemes & put them up in his pocket & went his way, leaving only what he blaterated againſt me: and did not only thus to *Browne*, but laboureth to recall all the bookes he had given forth, (which were many) before the ſight of *Brownes Lines*. And ſhortly after this he got a new Printer (who was ignorant of his former Schemes) to print him new: giving him an eſpeciall charge of the *outermoſt line newly graven* in the Plate, which indeed is *Brownes Verg line*: and then altering his book and craking of wonders in *Proſthaphæreſes*, he diſperſeth them by foures and ſixes. But ſee how it pleaſed God (who confoundeth the proud in their owne imaginations)

imaginations) to bring to light *Brownes* right and *Delamains* falſity. *Browne* himſelfe was preſent when the Plate was brought to the Printer, and heard the charge given concerning the new line: and ſince by *Brownes* friends have beene gotten diverſe of *Delamaines* former bookes and ſome Schemes of his Inſtruments, in none of all which that *great line of Browne* is to be found: And yet ſuch is *Delamaines* audacity (not knowing what can be ſhewn) that he ſtands to it to *Browns* face, that the ſaid *great line* was in the ſchemes in his former books. Wherfore we will (without ſtealing) borrow our Authors own words, *The window hath bin as yet cloſe, and darkneſſe poſſeſſeth the place, I now withdraw the curtaine that the ſunſhining light may appeare to expell thoſe miſts that have beene ſcattered, and by a true and ſincere medium remove that which* by Rich: Delamain *hath been falſely ſuggeſted.*

Well then, to come at laſt to a concluſion concerning the Inſtrument called the Circles of Proportion, as it is ſet forth, not having, as I have ſaid, the one halfe of my intentions upon it; nor with a ſecond moveable circle and a thread; but with an opening Index at the center (if ſo be that bee cauſe enough to make it to bee not the ſame, but another Inſtrument) for my part I diſclaime it: it may goe ſeeke another Maſter: which for ought I know, will prove to be *Elias Allen* himſelfe: for at his requeſt only I altered a little my rules from the uſe of the moveable circle and the thread, to the two armes of an Index.

And now moſt noble Gentlemen my Readers and Iudges, I humbly thanke you for the great patience you have ſhewed in hearing me alſo ſpeake for my ſelfe. I doe not requeſt of you any partiall reſpect or favour towards me at all: but onely what your wiſdomes ſhall ſee the ſimple honeſty of my cauſe doth deſerve. And what ſentence ſoever you ſhall be pleaſed to give herein; I will moſt ſubmiſſively, without any farther appeale, reſt in it. Onely I ſhall beſeech you to looke backe and conſider whether *R. D.* hath any the leaſt colour of ſhew for his ſo vile and baſe behaviour toward me, in ſcoffing, ſlandering, calumniating, back-biting, and exclaiming againſt me: contrary to all rules of charity and Chriſtianity, yea even of humanity and good manners. What wrong can he charge me, or indeed doth he charge me with, for which he may have a ſeeming ground of his ſo great malice? Was it becauſe ſo many yeares before I ever heard of his name, I prevented him in the invention of thoſe Inſtruments? That was the gift of God, and his proſpering my painfull ſtudy. Was it becauſe I have not made them more publick all this while? That was my modeſty. Was it becauſe I at laſt produced them to light? Neither was this my doing, but permiſſion

only. Was it for not giving way to him, when hee was pleaſed to lay hold upon both, to mount up with the wings of vain-glory by? I withſtood him not, nor once opened my mouth againſt him: but rather furthered him. And if underſtanding men, knowing his inabilities, and ſeeing the folly and ignorance he ſheweth in his Pamphlets, did even cry him downe, and almoſt with one conſent and voice acknowledge the true Authour: I ſought it not. Was it for not hindering *William Forſter* to publiſh the tranſlation, which with a great deale of labour hee had brought to an end: or not diſavowing it when it was printed? I neither had ſuch power over him: nor any reaſon at all to fruſtrate his long taken paines, for the ambition of another. Was it for making compariſons with him? I made none. Was it for my paines taken with him in teaching and inſtructing ſo ill natur'd a man? My gentleneſſe and good will deſerved better reſpect. Or was it for my ſo long and patient bearing his injurious reproaches, and unmannerly bravings of me? It was my meekneſſe, humility, and good nature. What other cauſe he could have againſt me, in the very ſtricteſt examination of my conſcience, I can find none. But hee had a mind to clambe, and thought my necke might make a fit ſtep for him to get up by. Indeed ſuch is the furious appetite of ſome wicked men, after their ambition and profit, that not the ſacredeſt tyes of Chriſtianity, friendſhip, or benefits received can with-hold them: but they will not ſticke even to cut the throats of their beſt deſerving friends, ſo that they may attaine to their intended purpoſes thereby I pray God ſuch be not his mind. for I heare he affecteth and is ambitious of public action and imployment: and ſomething he thinkes he muſt doe, that he may ſeeme ſomebody, and make himſelfe famous.

Concerning that he hath *in the behalfe of vulgar Teachers, and others.*

THere is yet a more fearefull Adverſary remayning, at the very thought of whom I am ſtriken with dread & trembling: which is your indignation and diſpleaſure moſt honoured and noble Gentlemen, and you moſt learned and expert Profeſſours of the Mathematicall Sciences: all whom this Challenger, as if his former injuries (moſt undeſerved on my part) were too little, in the higheſt ſtraine of his malice laboureth to exaſperate and incenſe againſt me: that *I with words downe-right and pernitious ſhould both glaunce upon many noble Perſonages with too groſſe, if not too baſe an attributes, by tearming them doers of trickes, and as it were to juggle; checking you groſſely, and abridging you of your liburties: and alſo by vilification ſhould*

ſhould ſtile the Teachers of Mathematics vulgar (common Teachers he would have them called) *ranking them with Iuglers, and teachers of trickes.* Farre be ſuch unreverent and unmannerly aſperſions againſt you from me, ever to approach neere my thoughts: much more to proceed forth of my mouth.

And I beſeech you obſerve with mee by what degrees his malicious ungratefulneſſe doth aſcend to the height of calumniation: firſt he ſaith, *my words, if they be truely ſcanned, rebound to the Nobility and Gentry:* then ſhortly after, *that they are downe-right in their plainenesſe.* which two accuſations ſeeme to imply a contradiction; if they needed *ſcanning*, and yet did but *rebound* even now: how are they ſuddenly become *downe-right in their plainenesſe?* And laſtly both openly in his Pamphlet, and in his rayling invectives againſt me in all company where he commeth, yea and (ſuch is his impudence) to my face, that mouth which hath very often implored my helpe, and ſubmiſſively acknowledged my courteſy, that very mouth, I ſay, hath not been aſhamed moſt ſlanderouſly to accuſe and charge me, That *in expreſſe words I ſhould call many of the Nobility and Gentry doers of trickes and iuglers:* which his bold and vile report, no doubt with many that know me not, nor the truth, but have given credite to his audacious afferſions, hath bred me much envy and diſcredite. Will it therefore pleaſe ſuch as have been ſo ill perſwaded, to vouchſafe to accept of a true and brieſe information: As I did to *Delamain* and to ſome others, ſo I did to *William Forſter:* I freely gave him my helpe and inſtruction in theſe faculties: only this was the difference, I had the very firſt moulding (as I may ſay) of this latter: But *Delamain* was already corrupted with doting upon Inſtruments, and quite loſt from ever being made an Artiſt: I ſuffered not *William Forſter* for ſome time ſo much as ſpeake of any Inſtrument, except only the Globe itſelfe; and to explicate, and worke the queſtions of the Sphære, by the way of the Analemma: which alſo himſelfe did deſcribe for the preſent occaſion. And this my reſtraint from ſuch pleaſing avocations, and holding him to the ſtrictneſſe of precept, brought forth this fruit, that in ſhort time, even by his owne skill, he could not onely uſe any Inſtrument he ſhould ſee, but alſo was able to delineate the like, and deviſe others: yet for all this my ſevere hand I ſaw him obliquely to glaunce his eye upon ſuch Inſtrumentary practices: whereat I being jealous, leſt I ſhould loſe my labour, and he his end, which was Art: I brake out into that admonition which in his Epiſtle Dedicatory to Sir *Kenelme Digby* he (I thinke in my very formall words) ſetteth downe. That the true way of Art is not by Inſtruments, but by demonſtration: and that it is a prepoſterous courſe of vulgar Teachers, to beginne with Inſtruments,

" ſtruments, and not with the Sciences, and ſo in ſtead of Artiſts,
" to make their Schollers onely doers of tricks, and as it were jug-
" lers: to the deſpight of Art, loſſe of precious time, and betraying
" of willing and induſtrious wits, unto ignorance and idleneſſe.
" That the uſe of Inſtruments is indeed excellent, if a man bee an
" Artiſt: but contemptible, being ſet and oppoſed to Art. And
" laſtly, that hee meant to commend to mee the skill of Inſtru-
" ments, but firſt he would have me well inſtructed in the Sciences.
Theſe words are to *Delamain* like a candle unto weake and rheuma-
tic eyes, his purblindneſſe cannot endure the brightneſſe of them, but
maketh him ſmart, and pricke, and vexe, and cry out, away with
this light it hurteth mine eyes, put it out: and meerely out of the
deteſtation of this light, and the diſproportion it hath to his weake-
neſſe, thoſe tragicall exclamations, wherewith his unlettered and
unmannerly Pamphlet is ſtuffed, have proceeded. Other Teachers
of theſe Arts, men of learning and skill, have (many of them) and
do daily acknowledge the truth, & ſeaſonableneſſe of this admoniti-
on, and are ſorry for the wrong done unto Art it ſelfe under colour of
me: Onely one *Richard Delamain* is found who forgetting truth,
gratitude, good manners, and very ſhame, doth bewray his gald back
by ſuch impatient winching. Little did I ever ſuſpect when I ſpake
theſe words privately at home, they ſhould be *ſcanned* with ſo un-
charitable and malicious a cenſure. Honoured Readers conſider I
pray you who it is that doth you wrong, and offereth you this con-
tumely: if it bee I who not ſo much as having the leaſt thought of
any of you, privately tutored my learner with a modeſt, gentle, and
ſeaſonable advertiſement: or if it be not *Richard Delamain* himſelfe,
that moſt inſolently, to cloke his owne unskilfulneſſe, and miſlea-
ding you in Art, and juggling, doth put upon your ingenuities that
baſe imputation, as if he had made ſome of you only *Doers of tricks
upon Inſtruments, and as it were taught you to juggle.* What his courſe
in teaching is I know not: but what his skill is I doe perfectly know.
And concerning my honorable eſtimation of you moſt worthy Gen-
tlemen, I doe unfeignedly glory in the behalfe of this our native
Countrey, that no Land under the cope of heaven, is more happy
with a gallant, and glorious flower of Gentry, and which is more
liberally enriched by nature with ingenuity, and all excellent endow-
ments both of wit, courage, and abilities of mind and body, and
hath more propenſe inclinations to all good, then this our ſweeteſt
and moſt indulgent mother of Great Britaine bringeth forth: only
if we can take care to plant in our minds the good ſeedes of vertue,
and knowledge: and not to neglect them to be overgrown (as the
beſt ground will) with the weedes of evill and contraryous habits.
whereunto on both ſides no one thing conduceth more then the
wiſe,

wiſe, or inconſiderate choice of Teachers, and Inſtructers. And then conſider I beſeech you what ſlight account this our glorious Challenger maketh of your worthyeſt endowments; that having ſo long ordered your ſtudies, diſpoſed of your times, and received your money, hath even in his owne conſcience done you ſo little good, that there being but the very name of *loſſe of time, jugling, and ignorance*, occaſionally mentioned, is himſelf firſt of all ready moſt unmanerly for your ingenuity, yet more unadviſedly for his owne credite, to taxe you therewith, and pinne it upon you. And will you moſt clearely ſee how he ſeeketh not your good, but your intollerable expence, for his ambition and vain-glory, and no good at all of yours? *His Ring* forſooth muſt be made *of Silver and Gold*: Braſſe belike is too baſe, or he feareth the waſting of it, leſt there may not be enough left to furniſh his face. And they muſt be *of ſixe feete diameter*, of which whether the monſtrouſneſſe, unprofitableneſſe, or exceeding charge will be the greater, I cannot readily tell. yet as if this were too little to exhauſt your eſtate, he hath a far more hideous device then all this, that is *a Cylinder of metall* (Silver ſure it muſt be) *three feet diameter, and of height ſufficient to receive* 100 *or more moveable rings, and as many fixed*, having within it ingens, and movements, and I know not what Automata (nor he neither) for the turning of thoſe rings, which by computation of skilfull workemen can hardly be performed for three hundred pounds: And when it is done, and you with a great deale of labour, can tell the uſe of it, you are not any way in Art the wiſer, or better by three ſingle pence: but in farre ſhorter time, and with much leſſe labour, you may be taught with a booke of twelve pence to worke and performe farre more, and more exactly, then by that monſtrous barrell *Delamain* himſelfe can ever be able to effect. And doe you not now (moſt noble Gentlemen) cry ſhame upon ſuch teachers; ſhame upon ſuch loſſe of time; ſhame upon ſuch profuſion of money; ſhame upon ſuch vile betraying of willing and induſtrious wits unto ignorance and idleneſſe; And many ſhames upon ſuch diſhoneſty, to ſet out in print againſt his owne knowledge, ſo many falſe propoſitions, and precepts purpoſely to abuſe the ignorance of his Readers, and that they may eſteeme him for ſome extraordinary and more then *a vulgar Teacher*.

Thus have I anſwered to *the three parts of his plea*: And I ſuppoſe that by this time you wonder as well as my ſelfe, what juſt cauſe there ſhould be of all his clamorous and malevolous inveighings againſt me. But you muſt give him leave to uſe his owne nature and manners. I am not the firſt, that have beene in this petulant manner provoked by him. Who indeed hath eſcaped him? The ſtirring hu-

mour of ſome is, that if they thinke they know any thing, they love to make a great noiſe, and raiſe a great duſt, till all become weary of them. Of this condition is our Challenger: whatſoever he hath, he muſt have it with ſuch a breath, that all the world ſhall heare of it, and all that come in his way ſhall ſuffer for it. England is too little, and his mothers tongue too barren (and yet if his mothers tongue were like his, it were copious enough) to yield him names, titles, inſcriptions, and expreſſions. But France, and Greece, and Latium muſt be raked and harrowed for him. to ſuch a height of courage and ſpirit ſelfe-admiration hath wrought him. The Aſſe in *Æſop* having got on a piece of the Lyons skinne beginneth to ſtrut and ruffle among the Beaſts. I am not only contemned by him (for that mattereth not much) But that incomparable Maſter *Henry Briggs* the mirrour of our age for excellent skill in Geometry, and therewith for exceeding meekneſſe, was ſo vilifyed and ſlighted by this niſle in a bagge, a little before his death, that the good old man forgetting his owne mild nature, at his laſt departure from London, being on horſe-backe for Oxford, and taking his leave of a friend, ſpake the laſt words, farewell, and tell *Delamain* from me that he is an abſurd fellow. and that we *may not wrong the dead, but give every man his due*, we muſt ſuffer him to poſſeſſe the legacie of ſo worthy a friend. Yea and Maſter *Gunter* too (whom he would ſeeme to admire) eſcapeth him not without a ſhrewd laſh. for in the beginning of his booke of the Horizontall quadrant pag: 3, he braveth him ſaying, *this Maſter* Gunter *delivers ſo* OBSTRVSELY *in his 66 page of the Sector, that if a man had not more fundamentall Mathematicall doctrine then his booke teacheth, he ſhould never attaine to it.* It is well for him Maſter *Gunter* is not now to give him a ſecond legacie: and to tell him a lyar had need of a good memory. for in that part of his *plea* which is *an anſwer upon his Quadrant* towards the end, hee writeth that *Joyners, and Carpenters, and ſchoole-boyes, and ſundry Gentlemen and others, having not had the leaſt aſſiſtance from any, but the direction of Maſter* Gunters *Booke alone, as upon oath they have been examined, have drawne the projection fully and compleatly.* which two places being ſo quite contrary are worth your comparing, that you may know *Richard Delamain* aright.

Which his uſage of Mr. *Briggs* and Mr. *Gunter* excuſeth him the more that he is ſo ſupercilious & ſtrange to others, who are alſo teachers of thoſe Arts, and farre more skilfull then himſelfe: diverſe of whom I have heard complaine and ſtomach at *Delamains* ſtanding ſo aloof, and keeping them off at ſuch diſtance from him, as not worthy of *his noble profeſſion*: & vehemently ſuſpect, that beſides his arrogancy, there was alſo a diffidence, and feare, leſt his ignorance might chance to bewray it ſelfe, as doth an Aſſe by his long eares.

I muſt

I Must now borrow a word or twaine with the Gentleman which writ the first Verses in the beginning of this Pamphlet, and stileth himselfe *a friend to the Inventer of the Logarithmes projected in circles.* Sir I see you are not disfavoured of the *Muses* and *Apollo:* your verse is good, and the conceipt well continued throughout: worthy of a better subject: or if you were pleased to play and shew your skill in so poore an argument, you might have spared me, who never offended you, and whom peradventure you know not so much as by sight. Did you ever heare me

——*deny it was found out by you?*

Did I ever tell you ——*it was mine owne?*

would I have professed and owned these and such slight toyes, I could have done it long before your *Inventer* had any ability of invention in this way. No I did nothing in publishing hereof: I onely gave way and permission. and it was not I that did addict it to my selfe: but his knowne worthlessenesse that did abdicate it from him. Neither is your argument of any force:

But if it were not thine, how durst thou say,
Thou wouldst augment the same another day?

Why, what can he not say? What dareth he not say, that may conduce to his vain-glory? And is it consequent, because he said *if the line were decupled*, therfore he first invented it? If you are as good at the Mathematics, as you are dexterous in making Meter; you cannot be ignorant that the breaking of the Circles one into many is no new invention; but is performed in the Circles of Proportion, as they were set out: wherein the Canon of Sines is broken into two circles, and the Canon of Tangents into foure. and I hope by the same reason I cou'd have broken them, or (if you will so call it) *augmented* or multiplyed them, into as many circles as I had desired. But whereas you Poetically jest at me,

Tis ten to one this will be challeng'd too,

I thinke you will prove your selfe to be a truer *vates* then you were aware of: though not by me (who have not esteemed such *minutiæ* worthy of me) but by *Thomas Browne* the Ioyner, whose indeed it is, and not your supposed *Inventers.* Sir you will be pleased to accept some reason in plain prose for your verse: and understand I doe you a favour to acknowledge you so farre.

And thus most honourable and noble Gentlemen having (as I hope even in your judgements also) vindicated my truth and honesty from such base petti-larcionary, as to steale his labours, and pilfer the wares of so poore a pack; and cleared my credite from the scurrilous and unmannerly calumniations & slanders, which he hath so unjustly endea-

endeavoured to faſten upon me: it will be high time to eaſe and free your patience from the trouble of ſuch idle altercations. And I humbly beſeech you, that if any where I ſeeme to take his injuries neerer to heart then in wiſedome I ſhould doe from ſo contemptible an adverſary; not to impute it to paſſion: but curteouſly to conſider the unſufferableneſſe of his moſt unworthy calumniations, and evill uſage of me. It may peradventure be expected that I ſhould alſo reade him a lecture of good manners: But I will not take any more paines in tutoring ſo ungratious a ſcholler. Onely I wiſh him to ſtudy over his own inſtruction. Yet this good advertiſement I will receive from him, *that I have worke enough at home: and that my calling inviteth me to ſpend my houres better*, then any more to trouble my ſelfe with anſwering him according to his folly.

Dixi.

WILLIAM OVGHTRED.

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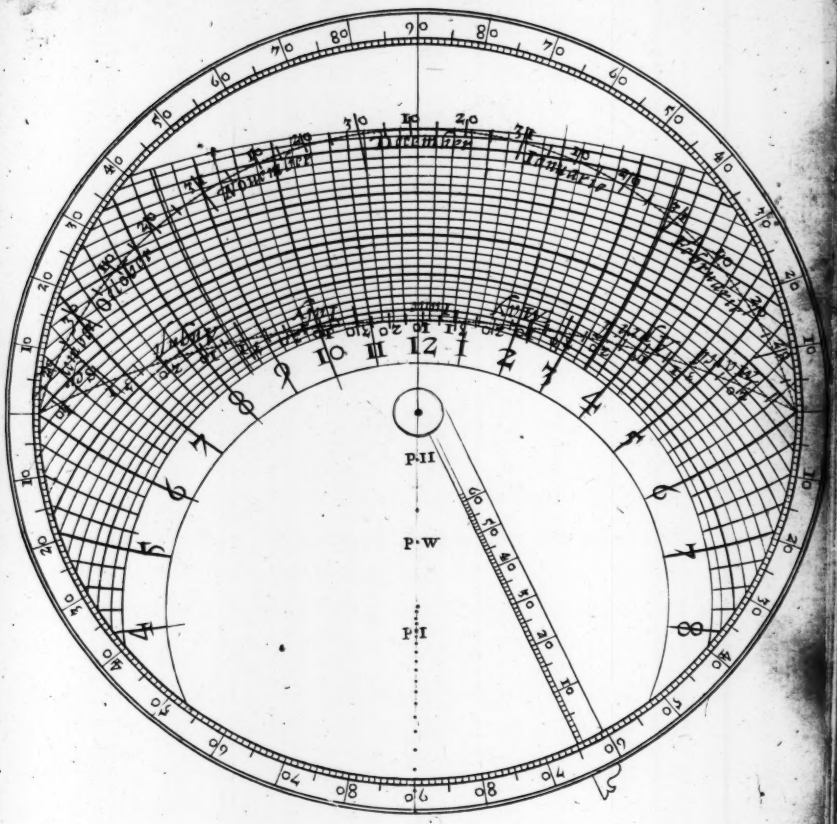
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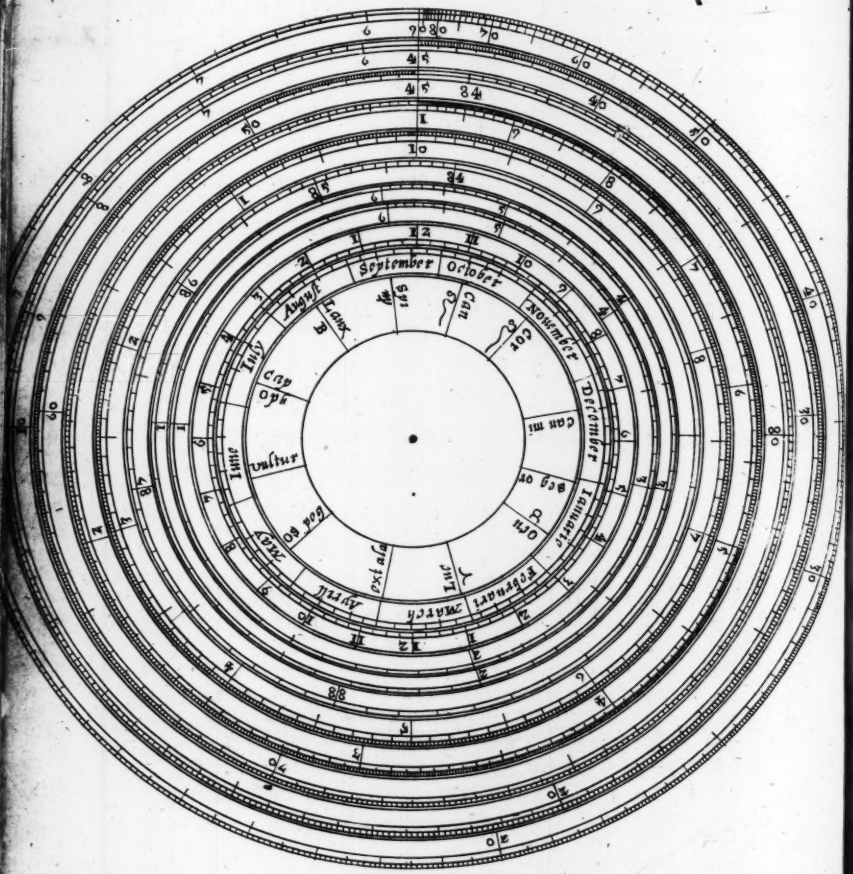


Thos that desire farther instructions in the use
of thes Instruments or other parts of the
Mathematiques may repaire to w Forster
at the Redbull ouer against st Clements church
yard with out Temylo bar

THE FIRST PART OF THIS BOOKE,

Shewing the vſe of the *Firſt ſide* of the Inſtrument, for the working of *Proportions both ſimple and compounded*, and for the ready and eaſie reſolving of queſtions both in *Arithmetique*, *Geometrie*, and *Aſtronomie*, by Calculation.

CHAP. I.

Of the Deſcription, and vſe of the Circles in this Firſt ſide.

1 THere are two ſides of this Inſtrument. On the one ſide, as it were in the *plaine of the Horizon*, is delineated the *proiection of the Sphære*. On the other ſide there are diuers kindes of Circles, divided after many ſeverall waies;

together with an *Index* to be opened after the manner of a paire of Compaſſes. And of this ſide we will ſpeake in the firſt place.

2 The *Firſt*, or outermoſt circle is of *Sines*, from 5 degrees 45 minuts almoſt, vntil 90. Every degree till 30 is diuided into 12 parts, each part being 5 min: from thence vntill 50 deg: into ſixe parts which are 10 min: a peece: from thence vntill 75 degrees into two parts which are 30 minutes a peeee. After that vnto 85 deg: they are not divided.

3 The *Second circle* is of *Tangents*, from 5 degrees 45 min: almoſt, untill 45 degrees. Every degree being divided into 12 parts which are 5 min: a peece.

4 The *Third circle* is of *Tangents*, from 45 degrees untill 84 degrees 15 minutes. Each degree being divided into 12 parts, which are 5 min: a peece.

5 The *Sixt circle* is of *Tangents* from 84 degrees till about 89 degrees 25 minutes.

The *Seventh circle* is of *Tangents* from about 35. min: till 6 degrees.

The *Eight circle* is of *Sines*, from about 35 minutes til 6 degrees.

6 The *Fourth circle* is of *Vnæquall Numbers*, which are noted with the Figures 2, 3, 4, 5, 6, 7, 8, 9, 1. Whether you vnderſtand them to bee ſingle Numbers, or Tenns, or Hundreds, or Thouſands, &c. And every ſpace of the numbers till 5, is divided into 100 parts, but after 5 till 1, into 50 parts.

The *Fourth circle* alſo ſheweth the *true* or *naturall Sines*, and *Tangents*. For if the *Index* bee applyed to any Sine or Tangent, it will cut the *true Sine* or *Tangent* in the fourth circle. And wee are to knowe that if the *Sine* or *Tangent* be in the *Firſt*, or *Second circle*, the figures of the *Fourth circle* doe ſignifie ſo many thouſands. But if the *Sine* or *Tangent* be in the *Seventh* or *Eight circle*, the figures in the *Fourth circle* ſignifie ſo many hundreds. And

if

if the *Tangent* bee in the *Sixt circle*, the figures of the *Fourth circle*, signifie so many times tenne thousand, or whole *Radij*.

And by this meanes the Sine of 23°, 30′ will bee found 3987 : and the Sine of it's complement 9171. And the Tangent of 23°, 30′ will be found 4348 : and the Tangent of it's complement, 22998. And the Radius is 10000, that is the figure 1 with foure cyphers, or circles. And hereby you may finde out both the summe, and also the difference of Sines, and Tangents.

7 The *Fift circle* is of *Æquall numbers*, which are noted with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 0; and every space is divided into 100 æquall parts.

This *Fift circle* is scarse of any use, but onely that by helpe thereof the given distance of numbers may be multiplied, or divided, as neede shall require.

As for example, if the space betweene 1^{00} and $1^{08\frac{33}{100}}$ bee to bee septupled. Apply the Index vnto $1^{08\frac{33}{100}}$ in the Fourth circle, and it will cut in the Fift circle 03476+; which multiplyed by 7 makes 24333 : then againe, apply the Index vnto this number 24333 in the Fift circle, and it will cut in the Fouth circle $1^{75\frac{68}{100}}$. And this is the space betweene 1^{00} and $1^{08\frac{33}{100}}$ septupled, or the Ratio betweene 100, and $108\frac{1}{3}$ seven times multiplied into it selfe.

And contrarily, if $1^{75\frac{68}{100}}$ bee to bee divided by 7 : Apply the Index vnto $1^{75\frac{68}{100}}$ in the fourth circle, and it will cut in the fit circle 24333 : which divided by 7 giveth 03476+. Then againe vnto this Number in the Fift circle apply the Index, and in the Fourth circle it will cut vpon $1^{08\frac{33}{100}}$ for the Septupartion sought for.

The reason of which Operation is, because this *Fift circle* doth shew the *Logarithmes* of Numbers. *For if the Index be applyed vnto any number in the* Fourth circle, *it will in the* Fift circle *cut vpon the* Logarithme *of the same number, so that to the* Logarithme *found you præfixe* a Caracte-

riſticall (*as Maſter* Brigs *termes it*) *one leſſe then is the number of the places of the integers propoſed* (*which you may rather call the* Graduall Number). So the Logarithme of the number 2 will bee found 0.30103. And the Logarithme of the Number 4316. will bee found 1.63949.

Numbers are multiplied by Addition of their Logarithmes: *and they are Divided by Subſtraction of their* Logarithmes.

8 In the middeſt among the Circles, is a *double Nocturnall* inſtrument, to ſhew the hower of the night.

9 The right line paſſing through the Center, through 90, and 45 I call the *Line of Vnitie*, or of the *Radius*.

10 That *Arme of the Index* which in euery Operation is placed at the Antecedent, or firſt terme, I call the *Antecedent arme*: and that which is placed at the conſequent terme, I call the *Conſequent Arme*.

CHAP.

CHAP. II.

Of the Operation of the Rule of Proportion: and alſo of *Multiplication*, and *Diviſion*.

1. *Theoreme.*

IF of three numbers given, the firſt divide the ſecond and the quotient multiply the third; the product ſhall be the fourth proportionall, to the three numbers given.

Theoreme. If of three numbers given, the ſecond divide the firſt, and the quotient divide the third; this later quotient ſhall be the fourth proportionall, to the three numbers given.

Neither is it materiall whether of the two numbers after the firſt be ſecond, or third.

2 And note that in *Reciprocal proportion*, that terme by which the queſtion is made; But in *Direct proportion* the terme that is *homogeneal* thereto, is the firſt terme, or the Antecedent of the firſt ratio.

3 And therefore out of theſe foundations thus layd, (if you rightly conceive the nature of the Logarithmes) doth follow *the finding out of the fourth proportionall by this Inſtrument*: whereof this is the Rule.

Open the Armes of the Inſtrument to the diſtance of the firſt, and ſecond number: then bring the Antecedent arme, or that which ſtood vpon the firſt number vnto the third, and ſo the conſequent arme, keeping the ſame opening, will ſhew the fourth number ſought for.

In which operation theſe foure things are diligently to be conſidered.

Firſt, in conſtituting the places of each number in the fourth circle; whether the figures written in the ſpaces doe ſignifie Vnites, Tenns, *or* Hundreds, *&c.*

Secondly, if that arme which ſheweth the fourth proportionall, doe reach beyond the line of the Radius; that then you doe account the fourth in a new circle or degree.

Thirdly, whether the fourth number ſought, ought to be greater, or leſſer then the third. For if a fourth number bee offered greater then the third, when it ſhould be leſſe, or leſſe then the third when it ſhould be greater; it is a ſigne that that number doth appertaine to a circle of another degree.

Fourthly, that looke what true diſtance was betweene the firſt and ſecond, that the ſame bee ſuppoſed betweene the third and the fourth, and alſo on the ſame part.

4 And for becauſe *Multiplication* and *Diviſion*, have a certaine implicite proportion: we will ſpeake of them in the firſt place.

5 *In Multiplication, As an Vnite is to one of the factores* (or numbers to be multiplied:) *ſo is the other of the factores, to the product.*

And the product of two numbers ſhall have ſo many places as there be in both the factores, if the leſſer of them exceede

exceede ſo many of the firſt figures of the product: But if it doe not exceede, it will have one leſſe.

6 *And in Diviſion, As the Diviſor is to an Vnite; ſo is the Dividend, to the Quotient.*

And the Quotient ſhall have ſo many places, as the Dividend hath more then the Diviſor, if the Diviſor exceede ſo many of the firſt figures of the Dividend: but if it doe not exceede, it ſhall have one place more.

7 Wherefore let this rule bee ſtill carefully kept in minde: that *In Multiplication the firſt terme of the implicite proportion is evermore* 1: And *in Diviſion, the firſt terme is the Diviſor.*

And thus much concerning the operation of Proportion, Multiplication, and Diviſion, I thought meete to admoniſh, leaſt hereafter in Multiplying, or Dividing, or ſeeking out a fourth proportional, wee be conſtrained to repeate the ſame things many times over.

8 An example of *Multiplication.* How many pence are there in $47^{li}. 9^{sh}$? For becauſe 1 ſhilling containes 12 pence, and 1 pound containes 20 ſhillings, that is 240 pence: you ſhall multiply 47 by 240, and 9 by 12, and then adde together the products.

In the firſt *Multiplication.*

1 . 47 : : 240 . 11280 .

For ſet the *Armes* of the *Index* at 1 and 47 in the fourth circle; and then bring the *Antecedent arme* (which ſtood at 1) vnto 240, and the *Conſequent arme* will ſhew 11280.

Againe in the ſecond *Multiplication.*

1 . 9 : : 12 . 108 .

For ſet the two *Armes* of the *Index* at 1 and 9 in the fourth circle; and bring the *Antecedent arme* unto 12, and the *Conſequent arme* will ſhew 108. Laſtly, adde together 11280 and 108 and the ſumme 11388 will be the number of pence contained in the ſaid ſumme of $47^{li}, 9^{sh}$.

9 An

9 An example of *Diuiſion.* How many pounds, and ſhillings are in 11388 pence? Divide 11388 by 240: the diuiſion is thus.

240 . 1 :: 11388 . 47^{li} —

For ſet the two Armes of the Index at 240, and 1 in the fourth circle: and then bring the Antecedent arme (which ſtood at 240) unto 11388; and the Conſequent arme will ſhew 47 and almoſt an halfe.

But how many ſhillings that exceſſe doth containe wil appeare, if firſt you finde by Multiplication that 11280 pence are contained in 47^{li}: which ſubducted from 11388 there will remaine for the exceſſe 108 pence. Afterwards by diuiſion you may ſeeke how many ſhillings are in 108 pence: the diuiſion is thus.

12 . 1 :: 108 . 9 .

For ſet the Armes of the Index at 12 and 1: then bring the Antecedent arme (which ſtood at 12) unto 108; and the Conſequent arme will ſhew 9.

9 *Any Fraction given may bee reduced into Decimal parts, thus.*

Set the Antecedent arme of the Index at the Denominator of the Fraction given, in the fourth circle, and the Conſequent arme at the Numerator, then keeping the ſame diſtance, bring the Antecedent arme unto 1, and the conſequent arme will ſhew the *decimal parts.*

So $\frac{90}{120}$ is 0$|$75. And $\frac{19}{63}$ is 0$|$306—

CHAP.

CHAP. III.

Now follow certaine examples of Proportion.

Example I.

IF 54 Elnes of Holland bee ſolde for 96 ſhillings, for how many ſhillings was 9 elnes ſold? The termes given are

54 . 96 : : 9 .

According to the 2 Chap. 3 Sect. Set one of the armes of the Index at the Antecedent terme 54 in the fourth circle, and the other arme at the conſequent terme 96: then keeping that diſtance, bring the Antecedent arme vnto 9; and the conſequent arme beyond the *line of the Radius* will ſhew 16 for the fourth proportionall, according to the *conſiderations* in the 2 Chap. Sect. 3.

Therefore

54 . 96 : : 9 . 16 .

are proportionals. And 16 ſhillings is the price of 9 Elnes.

Example II. If 108 buſhels of corne be ſufficient for a company of Souldiers keeping a Fort, for 36 dayes, How many dayes will 12 buſhels ſuffice that ſame number of Souldiers? The termes giuen are

108 . 36 : : 12 .

Set one Arme of the Index at the Antecedent terme 108 in the fourth circle, and the other Arme at the conſequent terme 12, (being mindfull of the conſiderations in the 2 Chap. 3 Sect.) then keeping that ſame diſtance,

bring the Antecedent arme vnto 36; and the conſequent arme will ſhew 4.

Therefore

108 . 36 : : 12 . 4 .

ſhall bee proportionals. And 4 is the number of dayes ſought for.

Example III. There is layd vp in a Fort ſo much corne as will ſuffice for 756 Souldiers which keepe that Fort, for 196 dayes: how many Souldiers will that ſame corne ſuffice for 364 dayes?

The Proportion is reciprocall, therefore the termes given are

364 : 756 : : 196 .

Set one Arme of the Index at the Antecedent terme 364, and the other Arme at the conſequent 196: and keeping the ſame diſtance, bring the Antecedent arme vnto 756; and the conſequent arme will ſhew 407 +: And therefore for ſo many Souldiers will the corne laid vp ſuffice for 364 dayes, or 13 moneths.

Example IIII. There is a Tower whoſe height I would meaſure with a Quadrant.

I take two Stations in the ſame right line from the Tower: and at either Station having obſerved the height through the ſights, I finde that the perpendicular cutteth in the nearer Station 28 degrees 7 minutes almoſt: and in the further Station 21 degr. 58 min. almoſt: and betweene both the Stations, the diſtance was 76 feet.

The Rule of meaſuring heights by two Stations is contained in theſe Theoremes.

Theor. As the difference of the Tangents of the arches cut in either ſtation, is to the diſtance betweene

tweene the ſtations; ſo is the Tangent of the leſſer arch, to the nearer diſtance from the Tower. Againe

Theor. As the Radius is to the Tangent of the greater arch; ſo is the nearer diſtance found, to the height.

And therefore becauſe by the 1 Chap. 6 Sect. the Tangents of the arches 28°,7′—, and 21°58′—are 5342, and 4032 whole difference is 1310; the proportions will be

First, 1310 . 76 : : tang. 21°, 58′— . 234 .

Wherefore 234 feet is the nearer diſtance.

Second Radius . tang. 28°, 7′— : : 234 . 125 .

Wherefore 125 feet is the height ſought for.

Example V. To finde the *Declination of the Sunne* the 9th day of May.

The place of the Sunne for every day, may be had here inough out of this Table, by Adding vnto the place of the Sun in the beginning of that moneth ſo many degrees, as there are daves paſt in that moneth: But if the number of degrees exceed 30, the exceſſe is to be accounted in the Signe next following.

Wherefore the 9th of May the place of the Sun is ♉ 20+9, that is ♉ 29: which is 59 degr. diſtant from the next Æquinoctiall point.

The place of the *Sunne, in the beginning of every Moneth.*		
Ianuary	♑	21
February	♒	22
March	♓	20
April	♈	21
May	♉	20
Iune	♊	19
Iuly	♋	18
Auguſt	♌	18
Septemb.	♍	18
October	♎	17
Novemb.	♏	18
Decemb.	♐	19

These things being knowne, the Rule is delivered in this Theoreme.

Theor. As the Radius is to the sine of the sunnes distance from the next Æquinoctiall point; so is the sine of the sunnes greatest declination, to the sine of the declination sought for.

The proportion will be

Radius . sine 59° :: sine 23°, 30′ . sine 19°,59′.

And so much is the Declination sought for.

Example VI. To finde the *Right ascension of the Sunne*, the 9th day of May.

Seeke the place of the Sunne for the day proposed in the former *Table*; and the Sunnes distance from the next Æquinoctial point, as in the former example.

These things being knowne, the Rule is by one of these two Theoremes.

Theor. As the Radius, is to the sine of the complement of the sunnes greatest declination; so is the tangent of the sunnes distance from the next Æquinoctiall point, to the tangent of the distance of the right ascension of the sunne, from the same Æquinoctial point. Or

Theor. As the tangent of the greatest declination of the Sunne, is to the Radius; so is the tangent of the declination of the sunne for the time proposed, vnto the sine of the right ascension of the sunne from the next Æquinoctial point.

The proportions will be either

Radius . sine 66° 30′ :: tang. 59° . tang 56° 46′.

Or, tang. 23°, 30′. Radius :: tang. 19°, 59′. sine 56°, 46′.

CHAP.

CHAP. IIII.

Of Continuall proportion, *Or of* Progression Geometricall.

1 THE *Ratio of a Progression is the quotient of the consequent terme devided by his antecedent.* And therefore in the Instrument it is *the distance taken betweene the termes in the fourth circle, by the opening of the Index.*

2 To *Double, Triple,* or *Multiply* how often soever *any Ratio given*, is nothing else but *so often to put together the said space or distance betweene the termes*; as is shewed in Chap 1. Sect. 7.

As for example, if the *Ratio* 60, to 65 be propосed to be *septupled.*

Set the Armes of the Index at 60, and 65: and then with the same opening, bring the Antecedent arme which was at 60, vnto 1, and the consequent arme will cut 1|08|33 + in the fourth circle, and 03476 + in the fift circle: this latter number being multiplied by 7 maketh 24333; vnto which number in the fift circle applying the Index, it will in the fourth circle cut 1|7e|2, which is the multiplied number sought for.

But because in a little Instrument, the arch cut in the fift circle, cannot be estimated exactly: and a small error in the beginning often repeated, by multiplying is made great: it is the most safe way, to take the *Logarithmes* of the termes of the *Ratio* out of the *Canon*, and to multiply them by the number given: As I have done in these examples.

Logarithmes taken out of the Canon,

100	2 . 0000000
104	2 . 0170333
105	2 . 0211893
106	2 . 0253059
106 1/3	2 . 0280287
107	2 . 0293838
107 1/3	2 . 0334285—
108	2 . 0334238—
108 1/3	2 . 0347621
108 2/3	2 . 0354297+
108 3/4	2 . 0364293—
109	2 . 0374265
110	2 . 0413927

Or if the *Canon* be wanting; you may come neerer the marke, if that firſt ſingle opening of the Index being kept, and the Antecedent arme ſet at 1; you transferre the Antecedent Arme, unto that place which the conſequent arme doth cutt; and the conſequent arme will cut the ſame ſpace *duplicated*. Then holding the conſequent arme in that ſame place, open the Antecedent arme unto 1. and afterward with that *duplicated opening*, bring the Antecedent arme to the *duplicated ſpace*, and the conſequent arme will cut the *ſpace quadrupled*. Thirdly, bring the Antecedent arme unto the *quadrupled ſpace* and the conſequent arme, keeping that *duplicated opening*, will cut the ſpace *ſextupled*. Laſtly, having againe taken a ſingle opening, bring the Antecedent arme unto the number, or *ſpace ſextupled*; and the conſequent arme will ſhew the *Ratio* ſought for *ſeptupled*.

And this manner of working may bee obſerved for as many Multiplications as you pleaſe of any *Ratio* given.

3. The *Ratio*, and *firſt terme being given, to continue the ſame unto any number of termes.*

Open the Armes of the Index, the one unto the Antecedent of the *ratio* given, and the other unto the conſequent: then the ſame opening being kept, bring the Antecedent Arme vnto the firſt terme given, and the conſequent arme will ſhewe the ſecond terme: againe bring the Antecedent arme vnto the ſecond terme found, and the conſequent arme will ſhew the third. After that bring the Antecedent arme unto the third terme found, and the conſequent arme wil ſhew the fourth. And in this manner you may proceed as farre as you pleaſe.

As for example, If a Progreſſion in the *ratio* 2 unto 5, beginning at 8, Or if a Progreſsion in the *ratio* 100 unto 108, beginning at 5, is to be inſtituted; the termes in either progreſsion will be as followeth.

1	2 . 5 :: 8 .	20
2		20
3		50
4		125
5		312$\frac{5}{}$
6		781$\frac{25}{}$
7		1953$\frac{125}{}$

1	100 . 108 :: 5 .	5$\frac{4}{}$
2		5$\frac{4}{}$
3		5$\frac{832}{}$
4		6$\frac{29856}{}$
5		6$\frac{8024448}{}$
6		7$\frac{346640384}{}$
7		7$\frac{934371161472}{}$

4 Theo-

4 *Theor:* The *ratio* of any former terme, in a row of continuall proportionals, vnto any of the termes following, is æquall to the *ratio* of the first terme unto the second, multiplied into it selfe according to the distance of that latter terme from the former.

As for example, The *Ratio* $5\frac{4}{}$ unto $6\frac{80244}{}^{8}$ which is the third terme from it, is æquall to the *ratio* of 100 vnto 108 triplicated, Or as the Cube of 100 unto the Cube of 108. Wherefore

5 The *Ratio*, and the *first terme of the Progression* being given, to finde out *any other terme required.*

First, multiply the *ratio* given into it selfe, according to the distance of the terme sought from the first terme, by the 2 sect: then say

As 1 *is to that multuple found; so is the first terme, to the terme sought for.*

Example, What will be the amount of 2 6^{li}, in 7 yeares by *Interest upon Interest*, after the rate of 20 pence in the pound?

Because 1 pound which is 20 shillings containeth 60 groates, and 20 pence conteine 5 groates, the rate of the *Interest* will bee 60 unto 65, Or 100 unto $108\frac{33}{}$ *; But the first terme is given 26, unto which there are to be acquired 7 other termes in continuall proportion.

First, by the 4 Chap: and 2 Sect. Let the *ratio* given be septuplicated, that is multiplied sevenfold into it selfe, which will be $1\frac{75}{}^{12}$. Then set the consequent arme of the Index vnto the *septuplicate number* $1\frac{75}{}^{12}$, and open the Antecedent arme vnto 1; and keeping the same opening

ning, bring the Antecedent arme unto 26; and the conſequent arme will ſhew 45$\frac{53}{125}$li the amount ſought for.

6 The *Ratio*, and *any other terme of the Progreſsion* being given, to finde the *firſt terme.*

Firſt multiply the Ratio given into it ſelfe according to the difference of the terme given from the firſt terme.

Then ſay

As that multuple is unto 1; *ſo is the terme given, unto the firſt terme.*

Example, what ſumme in 7 yeares did amount unto 45$\frac{53}{125}$li by Intereſt upon Intereſt after the rate of 100 unto 108$\frac{33}{100}$+?

Firſt the ratio being ſeptuplicat, by the 4 Chap. 2 Sect. will be 1$\frac{75}{125}$. Then ſetting the Antecedent arme of the Index to that ſeptuple 1$\frac{75}{125}$, open the conſequent arme unto 1: and keeping the ſame opening, bring the Arme unto 45$\frac{53}{125}$li: and the conſequent arme will ſhew 26, which was the ſtocke, or ſumme of money, from which that amount did ariſe.

7 The *Ratio*, the *Firſt terme*, and *any other terme of a Progreſsion* being given, to finde *how many places the terme given is from the firſt terme.*

Firſt ſay, As the firſt terme is unto 1; *ſo is the other terme given, unto the ratio multiplied into it ſelfe according to the diſtance of that terme from the firſt.*

Wherefore according to 1 Chap. 7 Sect. and 3 Chap. 2 Sect. by helpe of the fift circle, ſee how often the ratio given, is contained in that multuple found.

Example, In how many yeares did 26li, by Intereſt

upon Intereſt after the rate of 100 unto 108$\frac{33}{100}$ + increaſe unto 45$\frac{53\,12}{100}$li?

Firſt ſet the Antecedent arme of the Index at 26, and the conſequent arme at 1: and keeping the ſame opening bring the Antecedent arme vnto 45$\frac{53\,12}{100}$, and the conſequent arme will ſhew 1$\frac{7\,12}{100}$: to which in the fift circle anſwereth 24333. Then becauſe unto the conſequent terme of the ratio 108$\frac{33}{100}$ + there agrees in the fift circle 03476, divide 24333 by 03476, and the quotient will be 7, the number of yeares ſought for.

8 The *Firſt terme*, and *any other terme of the Progreſſion* being given to finde the *ratio of the Progreſſion*.

Firſt ſay, *As the firſt terme is unto* 1: *ſo is the other terme given, unto the ratio multiplied into it ſelfe according to the diſtance of that terme from the firſt.*

Wherefore according to Chap. 1. Sect. 7, by helpe of the fift circle, Let the multuple found be divided by the diſtance of the terme from the firſt.

Example, 26li by Intereſt upon Intereſt in 7 yeares amounted unto 45$\frac{53\,12}{100}$: what was the ratio of the Intereſt compared unto 100?

Firſt ſet the Antecedent arme of the Index at 26, and the conſequent arme at 1: and then keeping the ſame opening, bring the Antecedent arme unto 45$\frac{53\,12}{100}$: and the conſequent arme will ſhew 1$\frac{7\,12}{100}$: unto which in the fift circle anſwereth 24333 and this number being divided by 7 will give 03476 +: unto which agreeth in the fourth circle 108$\frac{33}{100}$ + the conſequent terme of the Intereſt ſought for.

9 *Two numbers being given* to finde *as many Midle proportionals betweene them as you will.*

Divide

Divide the diſtance of the greater number given from the leſſer in the fourth circle iuſtly taken, according to Chap. 1. Sect. 7, by helpe of the fift circle into equall ſegments, one more then are the number of Midle proportionals ſought for. All theſe ſegments added orderly to the firſt terme, doe diſtinguiſh the termes of the Progreſſion which you ſeeke.

Example, Let there be foure *Midle proportionals*, ſought out betweene 8 and $19\frac{656}{1000}$.

Apply the Index unto 8 in the fourth circle, and it will cut in the firſt circle 9039: alſo ſet the Index unto $19\frac{81}{100}$ in the fourth circle, and it will cut in the fift circle 2989; which number becauſe it reaches beyond the Vnite line, is indeede 12989, according to Chap. 1. Sect. 7, and ſo is the diſtance iuſtly taken. Then ſubducting 9039 from 12989, there will remaine 3950: which divided by 4 + 1, the quotient will give 790. wherfore 9039 + 790, ſcil. 9829 in the fift circle doth agree with $9\frac{6}{10}$ in the fourth circle, which is the *firſt midle proportionall*. And 9039 + twiſe 790, ſcil. 10619 in the fift circle doth agree with $11\frac{5}{10}$ in the fourth circle, which is the *ſecond midle proportionall*. And in this manner $13\frac{824}{1000}$, and $16\frac{5888}{10000}$ will be found the *third* and *fourth midle proportionals*.

10 *Theor.* If from the Ratio given, being multiplied in it ſelfe according to the number of termes, you ſubduct 1, and multiply the remainder, by the antecedent of the ratio. It will be

As the difference of the termes of the ratio, is unto the product; ſo is the firſt terme, to the ſum of the Progreſsion.

As for example if the *Ratio* of the Progreſsion be R to S: and the *difference* of R taken out of S be D. and the *firſt terme of the Progreſsion* a: and the whole *ſumme of the termes* Z it ſhall be,

D . rat. mul^{ta} — 1 in R :: α . Z .

Which is the very Theoreme it ſelfe expreſſed in *Symboles of words*: that it may more eaſily be fixed in the phantaſie. Which proportion alſo wee muſt conſider, doth hold both *alternly*, and *converſly*.

This Theoreme may otherwayes be expreſſed, by the equality which the product of the two midle termes hath to the product of the two extreames, thus

Ratio multiplicata — 1 in R in α = Z in D .

This manner of ſetting downe Theoremes, whether they be Proportions, or Equations, by Symboles or notes of words, is moſt excellent, artificiall, and doctrinall. Wherefore I earneſtly exhort every one, that deſireth though but to looke into theſe noble Sciences Mathematicall, to accuſtome themſelues unto it: and indeede it is eaſie, being moſt agreeable to reaſon, yea even to ſence. And out of this working may many ſingular conſectaries be drawne: which without this would, it may be, for ever lye hid as in this preſent proportion: becauſe it is

D . rat. mul^{ta} — 1 in α :: α . Z . wherefore

Rat. mul^{ta} — 1 in R . D :: Z . α . And

α . Z :: D . rat. mul^{ta} — 1 in R

Theſe are exceeding eaſie: but this following is more difficult, and requireth attention.

In the former *æquation* it was

Rat. mul^{ta} — 1 in R in α = Z in D

Now becauſe Rat. mul^{ta} — 1 in R in α,

and Rat. mul^{ta} — in R in α — R in α,

and Rat. mul^{ta} in R — R in α,

and

and Rat. mul^ta in $\alpha - \alpha$ in R,

are equall one to another, and alſo to Z in D, theſe æquations ſhall alſo be conſtarious.

$$\frac{\text{Rat. mul}^{ta} - 1 \text{ in R in } \alpha}{D} = Z .$$

and $\frac{Z \text{ in } D}{\alpha} =$ rat. mul^ta in R — R .

and $\frac{Z \text{ in } D}{R} =$ rat. mul^ta in $\alpha - \alpha$.

and $\frac{Z \text{ in } D + R \text{ in } \alpha}{R \text{ in } \alpha} =$ rat. mul^ta

And beſides theſe many more. The practiſe whereof I leave to the induſtry of the ſtudious Reader, eſpecially having delivered the whole Art of ſuch operations in my *Clavis Mathematica.*

Some of theſe I have occaſion to uſe in the ſections following.

11 *Therefore the* Ratio of a Progreſſion, *and the* firſt terme, *and the* number of termes *being given, to finde the* ſumme of the whole progreſſion.

For this operation the rule, or Theoreme laſt before ſerveth: for by it

$$\frac{\text{Rat. mul}^{ta} - 1 \text{ in R in } \alpha}{D} = Z .$$

Example. If an *Annuity* of 5^{li}, be detained 7 yeares, what will be the *amount* thereof by intereſt upon intereſt after the *rate* of 100 unto 108?

Now becauſe the *Amount is the ſumme of the Progreſſion, whereof the firſt terme is the annuitie*, Multiply the ratio into it ſelfe according to the number of yeares, by Chap. 4. Sect. 2, and it will be $1\frac{71382}{}$: from which if you ſubduct an vnite, there remaines $0\frac{71382}{}$, which multiplied by 100 maketh $71\frac{1382}{}$: then ſet the Armes of the Index at 1 and $71\frac{1382}{}$; and bring the Antecedent arme which ſtood at 1, unto 5 the firſt terme: and the conſequent arme will cut $356\frac{1915}{}$ (that is Rat. mul^{ta} — 1 in R in α) and laſtly this number being divided by 108 — 100, ſcil. by 8, the quotient will give $44\frac{6140}{}$, for the *ſumme of the whole progreſſion*: and ſo much is the *amount* ſought for.

12. The *Ratio*, the *Number of Termes*, and the *Summe of a Progreſſion* being given, to find the *firſt terme*.

By the *converſe* of the foregoing Theoreme, it is manifeſt, that

Rat: mul^{ta} — 1 *in* R . D :: Z . α .

The declaration in words was in the 10 Sect.

Example. If an *Annuitie* detained 7 yeares by Intereſt upon Intereſt, after the rate of 100 unto 108, did increaſe unto $44\frac{6140}{}^{li}$. how much was that Annuitie?

Multiply the *Ratio* into it ſelfe according to the number of yeares (*per cap. 4. ſect. 2.*) and the product will be $1\frac{71382}{}$: from which if you ſubduct an Vnite, there remaines $0\frac{71382}{}$: this being multipled by 100 doth make $71\frac{1382}{}$. Then ſay

$71\frac{1382}{} \cdot 8 :: 44\frac{6140}{} \cdot 5 \cdot$

for

for the Firſt terme: which was the *Annuitie* ſought for.

13 The *Ratio*, the *Firſt terme*, and the *Summe of the Progreſsion* being given, to find the *Number of termes*.

By the Theoreme in the 10. Sect. it was

α . Z :: D . rat. mul^{ta} — 1 in R.

Wherefore ſet the Antecedent arme of the Index unto the Antecedent terme of the ratio, and the conſequent arme unto the Summe of the Progreſſion: and with that ſame opening, bring the Antecedent arme unto the difference of the termes of the ratio; and the conſequent arme wil ſhew a number (that is Rat. mul. — 1 in R;) which if you divide by the Antecedent terme of the ratio, and unto the quotient adde an Vnite, you ſhall have the ratio multiplied into it ſelfe according to the number of termes. Therefore taking the diſtance betweene the termes of the ratio, with the armes of the Index, meaſure by helpe of the fift circle (*per Cap.* 4. *Sect.* 7.) how often that diſtance may be found in the multiplied ratio: for ſo many are the termes of the progreſſion.

Example. If an *Annuitie* of 5^{li} detained by Intereſt upon intereſt after the rate of 100 to 108, increaſed unto 44$\frac{6140}{1000}$^{li}. How many yeares was the *Annuitie* detained?

Set the Antecedent arme of the Index at 5, and the conſequent arme at 44$\frac{6140}{1000}$ and with the ſame opening bring the Antecedent arme unto 8, and the conſequent arme will ſhew 71$\frac{382}{1000}$ (that is Rat. mul^{ta} — 1 in R:) this number being divided by 100, will be 0$\frac{71382}{1000000}$: and if unto the quotient you adde 1, you ſhall have 1$\frac{71382}{1000000}$ (the ratio multiplied into it ſelfe according to the number of yeares:) unto which in the fift circle anſwereth 2338:

but

but unto 108 in the fift circle there answereth 0334. divide therefore 2338 by 0334; and the quotient will be 7, for the number of yeares sought for.

Or such questions may be more easily performed by this Theoreme, which the industrious Reader may by himselfe practise.

$$\frac{Z \text{ in } D + R \text{ in } \alpha}{R \text{ in } \alpha} = \text{rat. mul}^{ta}$$

14 *Theor.* If the summe of the whole progression be divided by the ratio multiplied into it selfe according to the number of termes, the quotient will be the first terme; and that summe given will be the last terme, of another progression, having the same ratio but one terme more.

15 And because a summe of mony the amount whereof in any number of yeares given, by Interest upon interest, doth equall an Annuitie so long detained, is equivalent to the same Annuitie; and the amount of an Annuitie is the summe of a Progression continued from that Annuitie. If therefore an Annuitie for any number of yeares be divided by the ratio multiplied into it selfe according to the number of yeares; the quotient will be the just price of an Annuitie to endure for so long. And because by the 10 and 11 Sect. it hath beene shewed that

$$\frac{\text{Rat. mul}^{ta} - 1 \text{ in } R \text{ in } \alpha}{D} = \text{to the amount.}$$

Therefore by the 14 Sect. $\frac{\text{Rat. mul}^{ta} - 1 \text{ in } R \text{ in}}{\text{Rat. mul}^{ta} \text{ in } D}$

will be equall to the price of an Annuitie in ready money, which shall be the Rule for the operation following.

Where-

Wherefore alſo

Rat.mul^ta — 1 in R in α = Rat.mul^ta in D in Pret.

which proportion is thus enuntiated in words.

Theor. If from the Ratio multiplied into it ſelfe according to the Number of yeares you ſubduct an Vnite, and the remainder be multiplied continually by the Antecedent of the Ratio, and the Annuitie it ſelfe: *And againe,* If the Ratio multiplied into it ſelfe, according to the number of yeares be multiplied continually by the difference of the termes of the Ratio, and by the Price: both thoſe products will be equall.

Example. An Annuitie of 5^li, to endure for 7 yeares, is to be ſold: what is it worth in ready money, after the Rate of 100 unto 108?

The Ratio multiplied into it ſelfe according to the number of yeares (*per Cap. 4 Sect. 2*) is 1,7138: ſubduct 1, and there will remaine 0,7138: which multiplied by 100 maketh 71,382. Then let the Antecedent arme of the Index at 1, and the conſequent arme 71,382: and keeping that ſame opening, bring the Antecedent arme unto 5; and the conſequent arme will ſhew 356,915; keepe this (for it is rat mul. — 1 in R in α / D). After that let the Antecedent arme of the Index at 1, and the conſequent arme unto the multuple ratio 1,7138; and with the ſame opening bring the Antecedent arme unto 8; and the conſequent arme will cut 13,7106, keepe this number alſo (for it is Rat.mul.in D). Laſtly, place the Antecedent arme of the Index at 13,7106, and the conſequent arme at 1; and with the ſame opening bring the Antecedent arme vnto 356,915: and the conſequent arme will ſhew 26,022li.

which is the iuſt Price of an Annuitie of 5^{li} in readie money.

16 And by the laſt præcedent Theoreme or Rule, alſo

$$\frac{\text{Rat mul. in D in Pret}}{\text{Rat.mul.}-1 \text{ in R}} = \alpha .$$

which Theoreme may bee enuntiated in words as was there ſhewed.

Example. An Annuitie for 7 yeares is bought for $26\frac{103}{125}$li. after the rate of 100 unto 108, by Intereſt upon intereſt: how much was that Annuitie?

The Ratio multiplied into it ſelfe for the number of 7 yeares (*per Cap.* 4, *Sect.* 2) is $1\frac{71}{125}$: which multiplied continually by 8, and by the price, doth make $356\frac{91}{125}$. Divide this number found, by $71\frac{82}{125}$, (which is the *multiple ratio*) it ſelfe leſſe by an Vnite, and multiplied by 100: and the quotient will be 5^{li}, the Annuitie ſought for.

17 Alſo by *Rationation* from that præcedent rule will follow this propoſition

$$\frac{\text{R in } \alpha}{\text{R in } \alpha - \text{Pret. in D}} = \text{Rat. mul}^{ta}.$$

which is thus enuntiated in words.

Theor. If the product of the Antecedent of the ratio multiplied by the Annuitie be divided by it ſelfe, being diminiſhed by the product of the difference of the termes of the ratio multiplied by the Price: the quotient will bee equall to the ratio multiplied into it ſelfe according to the number of yeares. As

If the ratio be 100 unto 108; the Annuitie 5^{li}: and the price thereof $26\frac{103}{125}$li. Set the Antecedent arme of the

the Index at 1, and the conſequent arme at 8, the difference of the termes of the ratio: and with the ſame opening bring the Antecedent arme unto 261$\frac{2}{3}$2 —: and the conſequent arme will ſhew 208$\frac{256}{}$: which ſubducted from 500, there will remaine 291$\frac{744}{}$, for the *diviſor*. Set therefore the Antecedent arme of the Index at the *diviſor* 291$\frac{744}{}$, and the conſequent arme at 1: and with the ſame opening, bring the Antecedent arme unto the *dividend* 500: and the conſequent arme will ſhew 1$\frac{718}{}$, which is the *ratio* multiplied into it ſelfe according to the number of yeares. And by this number ſo found it will be eaſie (by helpe of the fift circle) the *ratio* of the Intereſt being given, to finde the continuance of the Annuitie.

Example. An Annuitie of 5^{li}, was bought for 26$\frac{0}{}$1^{li}—, after the rate of 100 unto 108: how many yeares is it to laſt?

Firſt ſeeke out the ratio multiplied into it ſelfe according to the number of yeares, which will be 1$\frac{718}{}$, according as was even now ſhewed in this Sect. to this in the fift circle there anſwereth 2338: but unto 108 there anſwereth in the fift circle 0334. Divide therefore 2338 by 0334; and the quotient will be 7 for the number of yeares ſought for.

CHAP. V.

Of the Quadrating, *and* Cubing of Numbers, Sides or Rootes: *and of the* Extraction of the Quadrate, *and* Cubic side, *or* roote, *out of* Numbers, *or* Powers *given.*

1 IF a number, side, or roote be multiplied into it selfe; the product will be a Quadrat. And if a quadrate bee multiplied into his owne side, or roote, the product will be a Cube.

Wherefore 1 . N :: N . Q .

and 1 . N :: Q . C .

2 If therefore a number be given to be Quadrated *Set the Antecedent arme of the Index at* 1. *and the consequent arme at the number given: then with the same opening bring the Antecedent arme to the number given; and the consequent arme will shew the Quadrat thereof.*

And the number of figures in a Quadrat of a single root (or which doth not exceede 9) is easily found out by those Rules, that have beene before delivered concerning Multiplication. But if a side or root consist of more figures then one; for each figure after the first it acquireth two more places of figures. And if any of the figures of the root given be decimal parts, cut off from the Quadrat found, twice so many of the last figures for decimals.

Example 1. The Quadrat of the side 7 is required. Say

1 . 7 :: 7 . 49

Set therefore the Antecedent arme of the Index at 1, and the consequent arme at 7; and with that opening bring

bring the Antecedent arme unto 7; and the conſequent arme will ſhew 49 which is the Quadrat ſought for.

Example II. The Quadrat of the ſide 57 is required.

Set the Antecedent arme of the Index at 1; and the conſequent arme at 57: then with that ſame opening bring the antecedent arme unto 57; and the conſequent arme will ſhew 3249, which is the Quadrat ſought for, conſiſting of foure places.

Example III. The Quadrat of the ſide, or root 570 is required.

Having found as before, 3249 for the quadrat of the ſide 57: put thereunto two circles: and it will bee 324900, the quadrat ſought for.

Example IIII. The quadrat of the ſide 574 is required.

Set the Antecedent arme of the Index at 1, and the conſequent arme at 574; and with the ſame opening bring the Antecedent arme unto 574; and the conſequent arme will ſhew 329476, the quadrat ſought for conſiſting of ſixe figures: but the two laſt figures cannot at all be diſcerned by the Inſtrument.

3 If a number be given to be *Cubed.*

Set the Antecedent arme of the Index at 1, and the conſequent arme at the number given; and with that ſame opening, bring the Antecedent arme unto the number given; and the conſequent arme will ſhew the Quadrat; then bring the Antecedent arme unto the Quadrat, and the conſequent arme with that ſame opening will ſhew the Cube of that ſide given.

The number figures in a *Cube* of a ſingle ſide, or root which doth not exceede 9, is eaſily found by that which hath beene before delivered concerning Multiplication:

But if the ſide, or root conſiſt of more figures then one; for each figure after the firſt it obtaineth three more places of figures. And if any of the figures of the root given be *decimal parts*, cut off from the *Cube* found thrice ſo many of the laſt figures for *decimal parts*.

Example. The Cube of the ſide, or root 7 is required.

Say, 1 . 7 :: 7 . 49 .

againe 1 . 7 :: 49 . 343 .

Set therefore the Antecedent arme of the Index at 1, and the conſequent arme at 7; and with that opening bring the Antecedent arme unto 7, and the conſequent arme will ſhew 49 the quadrat thereof: Then ſet the Antecedent arme at 49; and the conſequent arme (with that firſt opening) will ſhew 343, which is the deſired Cube of the ſide propoſed.

Another example, The Cube of the ſide, or root 57 is required.

Set the Antecedent arme of the Index at 1, and the conſequent arme at 57; and with that ſame opening bring the Antecedent arme unto 57; and the conſequent arme will ſhew the quadrat 3249. Then ſet the Antecedent arme at 3249; and the conſequent arme keeping the former opening will ſhew 185193, which is the required Cube of that ſide propoſed, conſiſting of ſix places: but the two laſt figures cannot be known by the Inſtrument.

Example III. The Cube of the ſide 570 is required.

Hauing found as before the Cube of the ſide, or root 57 to be 185193: put thereunto three circles; and it will be 185 193000 the Cube ſought for.

Examples of greater Cubes, it will be needleſſe to ſet downe.

4. The

4 The *Extraction of the Square, or Quadrat root, or ſide,* is done by helpe of the fift circle, after this manner.

Set the Index at the Quadrat propoſed; and of that number which it cuts in the fift circle, take halfe: then ſet the Index at that halfe; and it will ſhew in the fourth circle, the ſide, or root ſought for.

But you muſt know that if the number which is the Quadrat propoſed, have onely two places of Integers, the ſide, or root conſiſteth of one figure. But if it have more places of Integers, dividing them by 2, the quotient will give the true number of figures in the root, if it meaſure it exactly; or one leſſe then the true number if any thing remaine.

Example I. The ſide, or root of the Quadrat 49 is required.

Set the Index at 49 in the fourth circle, and it will cut in the fift circle 6902; indeede 1.6902 having the *graduall number* 1 præfixed, becauſe 1 in the fourth circle ſignifieth 10, one *circuition* thereof being finiſhed: the halfe whereof is 0.8451. Then ſet the Index at 0.8451, in the fift circle; and it will cut in the fourth circle 7, the ſide, or root ſought for.

Example II. The ſide, or root of the quadrat 3249 is required.

Set the Index at 3249 in the fourth circle, and it will cut in the fift circle 5118; indeede 3.5118 prefixing the *graduall number* 3, becauſe 1 in the fourth circle ſignifieth 1000, three *circuitions* thereof being finiſhed: the halfe whereof is 1.7559. Then ſet the Index at 7559, omitting the prefixed *graduall number* 1; and it will ſhew in the fourth circle 57 the ſide ſought for, conſiſting of two figures, becauſe 1 in the fourth circle ſignifieth 10.

Example

Example III. The ſide of the quadrat 329476 is required.

Set the Index at 329476 in the fourth circle, and it will cut in the fift circle 5178; indeede 5.5178 prefixing the *graduall number* 5, becauſe 1 in the fourth circle ſignifieth 100000, five *circuitions* thereof being paſt over; the halfe whereof is 2.7589; Then ſet the Index at 7589, omitting the *graduall number* 2 prefixed thereto; and it will ſhew in the fourth circle 574 the ſide, or root ſought for, conſiſting of three figures, becauſe 1 in the fourth circle doth ſignifie 100.

5 The *Extraction of the Cubic roote, or ſide* is done by helpe of the fift circle after this manner.

Set the Index at the Cube propoſed; and that number which it cuts in the fift circle divide by 3. *Then ſet the Index at that third part, and it will ſhew in the fourth circle the ſide, or roote ſought for.*

And you muſt know that if the Cube propoſed have onely three places of Integers, the ſide, or roote thereof conſiſteth of one figure: But if it have more places of Integers, divide them by 3, the quotient will give the true number of the figures of the Root, if it meaſure it exactly; or one leſſe then the true number if any thing remaine.

Example I. The ſide, or roote of the Cube 343 is required.

Set the Index at 343 in the fourth circle, and it will cut in the fift circle 5353; indeede 2.5353 prefixing the *graduall number* 2, becauſe 1 in the fourth circle doth ſignifie 100, two *circuitions* therof being paſt over: the third part whereof is 8451. Then ſet the Index at 8451, and it will ſhew in the fourth circle 7, the ſide ſought for.

Example

Example II. The ſide, or roote of the Cube 185193 is required.

Set the Index at 185193 in the fourth circle; and it will cut in the fift circle 2677; indeede 5.2677 prefixing the *graduall number* 5, becauſe 1 in the fourth circle doth ſignifie 100000, five *circuitions* thereof being paſt over: the third part whereof is 1.7599. Then ſet the Index at 7599, omitting the prefixed *graduall number* 1; and in the fourth circle it will ſhew 57, the ſide, or root ſought for, conſiſting of two figures, becauſe 1 in the fourth circle doth ſignifie 10.

Examples of greater Cubes it will be needleſſe to ſet downe.

CHAP. VI.

Of Duplicated, *and* Triplicated proportion.
And first of Duplicated proportion.

Theoreme.

LIke *Plaines are in a Duplicated ratio*, that is, *As the Quadrats of their homologal sides*. And therefore questions in the which the *sides of like plaines are compared*, doe appertaine vnto this place.

And it is to be noted, that if three numbers be given, in which *As the quadrat of the first is unto the quadrat of the second; so ought the third to be unto a number sought for.* Let it be thus done, *As the first number, is to the second, so is the third to a fourth:* And againe *As the first number, is to the second, so is the fourth now found, to the number sought for.*

Example I. There are two like rectangle *Areæ*, or *plaines*, the length of the greater, is 40 feete, the length of the lesser 24 feet: each of them paved with paving tiles; the greater hath 1200 tiles: how many shall the lesser have?

The *Areæ*, or *plaines* are one to the other, as the quadrats of the longitudes given. And the proportion is direct. Say therefore

1600 (Q: 40) . 576 (Q: 24) :: 1200 . 432.

which is the number of tiles contained in the pavement of the lesser Areæ, or plaine.

Example II. How many Acres of woodland measured with a Perch, of 18 feete, are there in 73 Acres of champane land, measured with a Perch of 16½ feete?

The

The measures given (18, 16½) being reduced into their leſt termes, are as 12 unto 11 : and the quadrats of theſe numbers, are, 144, and 121. And the Proportion is Reciprocall. Say therefore

144 (Q: 12) . 121 (Q: 11) :: 73 . 66 . 61 49/144

and ſo many are the Acres of Wood land.

Of *Triplicated proportion.*

3 *Theor.* Like Solids are in a Triplicated ratio, *that is*, As the Cubes of their homologal ſides. *And therefore queſtions in which the ſides* of like ſolids are compared, *doe appertaine unto this place.*

4 *If three numbers be given in the which*, As the Cube of the firſt is to the Cube of the ſecond; ſo is the third number to a number ſought for. *Let it be thus done*, As the firſt number is to the ſecond; ſo is the third to a fourth: *Againe*, As the firſt is to the ſecond; ſo is the fourth now found unto a fift. *And thirdly*, As the firſt is to the ſecond; ſo is that fift to the number ſought for.

Example. If 43/100 lib. of gunpowder, ſuffice to charge a Gun, whereof the concave diameter is inch 1½. How many pounds of powder will ſuffice to charge a Gunne, whoſe concave diameter is 7 inches?

The capacities are one to another, as the Cubes of the diameters. And the proportion is direct. Say therefore

3375/1000 (C: 1½) . 343 (C: 7) :: 0143 . 4 31/100 +

wherefore 4 31/100 lib. of Gunpowder, will bee needfull to be had.

Another example, $43\frac{17}{343}$ lib. of Gunpowder are ſufficient to charge a Gun, whoſe diameter in the concave is 7 inches: now there is another ſort of Gunpowder, much more ſtrong and forcible, that is in ſtrength unto the former, as 5 vnto 2: How much of this ſtronger powder, will ſuffice to charge a Gun of 4 inches diameter?

Here are two operations: the firſt ſeekes out, how much of that ſtronger powder ſufficeth to charge a Gun of 7 inches diameter: and the proportion is reciprocall, that is

$$5 \cdot 2 :: 43\frac{17}{343} \cdot 15\frac{48}{343}$$

The ſecond operation is like that in the former example.

$$343 \{C: 7) 64 (C: 4) :: 15\frac{48}{343} \cdot 2\frac{288}{343}$$

CHAP.

CHAP. VII.

Concerning the Measuring *of* Circles, Cones, Cylinders, *and* Sphæres.

1. ARchimedes in a peculiar Treatise found the proportion of the Diameter of a circle to the Circumference to bee a very smale deale greater then of 7 unto 22: And of late *Ludolph Van Ceulen* insisting in the same steps of *Archimedes*, hath more precisely found it to be of 1 vnto 3,14159265358979793 but for our Instrument it will be sufficient to take the ratio of 1 vnto 3,1416, Or of 0,3183+ unto 1: leaving the diligent practizer, to more exactnesse, if he please to use his Pen.

And note that the Rules following are set downe in proportions, to be wrought as hath been taught in Chap. 2, Sect. 3. Wherein

D, or Diam. signifieth the Diameter.

Dq, or Q. Diam. the Quadrat of the Diameter.

Dc, or C. Diam. the Cube of the Diameter.

R, or Rad. the Radius, or Semidiameter.

P, or Perif. the Periferie, or Circumference.

Pq, the Quadrat of the Periferie.

Long. the length.

L the side, or latus

Alt. the altitude.

*, sheweth that the two magnitudes betweene which it is set, are to be multiplied together.

In a Circle.

2. *The* Diameter *of a circle being given, to finde the* Periferia. *Say,*

7 . 22, Or 1 . 3.1416 :: Diam . Perif .

Example. A circle is given, the Diameter whereof is 12, I would know the circumference, or Periferia of it. Say,

1 . 3.1416 :: 12 . 37.6992 .

3 *The* Periferia *of a circle being giuen, to finde the* Diameter. *Say,*

22 . 7, Or 1 . 0.3183 + :: Perif . Diam .

4 *The* Diameter *of a circle being giuen, to finde the* Area. *Say,*

7*4 . 22, Or 1 . 0.7854 :: Q: Diam. Area .

Or els 1 . 3.1416 :: Q: Rad . Area .

Example. A circle is giuen, the diameter whereof is 12, I would know the content, or Area of it. Say,

1 . 0.7854 :: 144 (Q: 12) . 113.0976 .

Or 1 . 3.1416 :: 36 (Q: 6) . 113.0976

5 *The* Area *of a circle being giuen, to finde the* Diameter. *Say,*

22 . 7*4, Or 1 . 1.27324 :: Area . Q: Diam .

Example. A circle is giuen, the content whereof is 113.0976, I would know the Diameter of it. Say,

1 . 1.27324 :: 113.0976 . 144 .

6 *The*

6 *The* Periferia *of a circle being giuen, to finde the* Area. *Say,*

22*4 . 7, Or 1 . 0,0795775 :: Q: Perif . Area .

7 *The* Area *of a circle being giuen, to finde the* Periferia. *Say,*

7 . 22*4, Or 1 . 12,566637 :: Area , Q: Perif .

In a Cone.

8 *The* ſide of a right Cone, *and the* Diameter of the baſe *being giuen, to find the* Superficies. *Say,*

7 . 22, Or 1 . 3,14116 :: ½ D*L . Superf .

Example. A Cone is giuen, whereof the ſide is 18, and the diameter of the baſe 12, I would know the ſuperficies of it. Say,

1 . 3,14116 :: 108 (½ D*L) . 339,2928 .

9 *The* Axis, *or* height of a right Cone, *and the* Diameter of the baſe, *being giuen, to find the* Soliditie. *Say,*

7*4 . 22, Or 1 . 0,7854 :: Dq in ⅓ Axis . Soliditie .

Example. A Cone is giuen, whereof the Axis is 18, and the Diameter of the baſe 12, I would know the Solidity. Say,

1 . 0,7854 :: 864 (Dq in ⅓ axis) . 678,2856 .

In

In a Cylinder.

10 *The* side of a right Cylender *and the* Diameter *being given, to finde the* Superficies. *Say,*

7 . 22, Or 1 . 3,1416 : : Diam : × axem . Superfic.

11 *The* Side of a right Cilinder, *and the* Diameter *being given, to finde the* Soliditie. *Say,*

7 × 4 . 22, Or 1 . 0,7854 : : Dq × L . Soliditie.

12 *The* Side of a right Cylinder, *and the* Circumference P, *being given, to finde the* Soliditie. *Say,*

22 × 4 . 7, Or 1 . 0,07957747 : : Pq × L . solidit.

In a Sphære.

13 *The* Axis, *or* Diameter *of a Sphære being given, to finde the* Superficies. *Say,*

7 . 22, Or 1 . 3,1416 : : Dq . Superficies.

14 *The* Superficies *of a Sphære being given, to finde the* Axis. *Say,*

22 . 7, Or 1 . 0,318310 : : Superfic . Dq .

15 *The* Segment *of a Sphære being given, to finde the* Superficies. *Say,*

7 . 22, Or 1 . 3,1416 : : $\left.\begin{matrix}\text{Q: chord}\\ \text{of } \frac{1}{3}\text{Segm.}\end{matrix}\right\}$. Superfic :

16 *The*

16 *The* Axis, *or* Diameter *of a Sphære being giuen, to finde the* Soliditie. *Say,*

7×6 . 22, Or 1 . $0\frac{5}{236}$ :: Dc . Soliditie .

Example. A Sphære is giuen, whereof the Axis is 12, I would know the ſolidity of it. Say,

1 . $0\frac{5}{236}$ :: 1728 (Dc) . $590\frac{5}{208}$.

17 *The* Soliditie *of a Sphere being giuen, to finde the* Axis. *Say,*

22 . 7×6, Or 1 . $1\frac{90}{986}$ :: Soliditie . Dc .

Example. A Sphære is giuen, the Solidity whereof is $590\frac{5}{208}$, I would know the Axis thereof. Say,

1 . $1\frac{90}{986}$ :: $590\frac{5}{208}$. 1728 (Dc) .

18 *A* Segment *of a Sphere being giuen to finde the* Soliditie. *Say,*

First. *As the altitude of the other Segment, is to the altitude of the Segment giuen: ſo is that altitude of the other Segment increaſed by halfe the Axis, vnto a fourth.* Then againe ſay, *As* 7×3 *is to* 22, *Or as* 1 *is to* 1047 2: *ſo is the product of the quadrat of halfe the chord of the Periferia of that Segment, multiplied by that fourth, to the Soliditie.* Viz.

7×3 . 22, Or 1 . $1\frac{5}{0472}$ :: $\frac{Q:\frac{1}{4}\ chordæ}{in\ quartam}$. Solidit.

19 For note that a Sphære, is equall to two Cones, having their height and the diameter of their baſe, the ſame with the Axis of the Sphære. Or which is all one, A Sphære is two third parts, of a Cylinder, having the height and the diameter of the baſe the ſame with the Axis of the Sphære.

CHAP. VIII.

Concerning *Plaine*, and *Solide Measures*.

1 THe diuiding of the *Carpenters ruler* into Inches, and halfe, and quarters, and halfe quarters o Inches, that is of euery Inch into eight parts, is most inartificiall, and vnfit for measuring, by reason of the manifold denominations, which must be brought into one, and is hard to bee done of them that are vnskilfull. I would wish therefore that euery Inch were diuided into 10 parts, or rather that the foot were diuided into 100 parts, which is best of all: for then there will neede no reduction. And all other diuisions must bee reduced vnto this, by these Rules following.

2. If the measures be taken vpon a Ruler diuided into Inches and tenth parts of an Inch, first take out all the whole feet, and then diuide the Inches remaining, with their decimall parts if there be any by 12.

Example. How many feet and decimall parts of a foot, are in Inches $17\frac{3}{10}$?

First take out the whole foot which is 12 Inches, and there will remaine Inches $5\frac{3}{10}$: which being diuided by 12, you shall haue 442 thousand parts almost: wherefore Inches $17\frac{3}{10}$, is feet $1\frac{442}{1000}$ ——. And contrariwise, feet $1\frac{442}{1000}$ —, shall be reduced into Inches $17\frac{3}{10}$, being multiplied by 12.

3. If the measure bee taken vpon a Ruler diuided into inches and halfe quarters, that is each inch into 8 parts, First you must reduce the eight parts into decimals of Inches, by diuiding the number of parts given by 8 the Denominator thereof: and afterward by the former Rule,

Rule, reduce the inches, and decimall parts, into decimall parts of a foot.

Example. How many feet, and decimall parts of a foot, are in 7 inches, and 5 eight parts?

Firſt diuide the 5 eight parts by 8, and you ſhall haue 625 thouſand parts: which being put to 7 inches, will make inches $7\frac{625}{}$: Again diuide theſe by 12, as was ſhewed in the former rule: and the whole meaſure will be feete $0\frac{635}{}$. And contrariwiſe feet $0\frac{635}{}$, will be reduced into inches $7\frac{625}{}$, being multiplied by 12.

4. I muſt aduiſe all thoſe that haue occaſion, to meaſure Plaines, or Solids, to make themſelues very perfect in this kind of Reduction (becauſe moſt Rulers they ſhall ordinarily meet withall, are diuided into inches, and halfe quarters) which will be very eaſie to them, if they doe but remember, that *In diuiſion the firſt terme of the proportion implyed is the Diuiſor it ſelfe: but in Multiplication, the firſt terme is euermore* 1. as hath beene ſhewed in Chap: 2, Sect: 7. And therefore, preſuming on the diligence of the Practiſer herein, I ſhall not neede in this kind of meaſuring, to ſpeake any more of inches, but of feet and decimall parts of feet, as if the Ruler were ſo diuided.

Of *Plaine meaſures.*

5. *A Parallelogram*, or foure ſided rectangle Superficies, being propoſed, to find the *length of a Superficiall foote.*

Take with your Ruler the breadth thereof in feet, and decimals of a foot: and by the breadth ſo taken diuide 1. the quotient ſhall be the length of a ſuperficiall foot.

Example. A boord is feet $1\frac{117}{}$ broad, how much thereof will make a foot?

Diuide 1 by $1\frac{17}{}$, the quotient will bee $0\frac{855}{}$ almoſt: ſo much ſhall the length of a foot be, which multiplying the parts by 12, will giue inches $10\frac{26}{}$. And againe thoſe parts multiplied by 8, will giue 2 eight parts of an inch.

Example, II. In tileing, or healing they vſe to reckon by the Square, which is 10 foot euery way, in all 100 feet. There is roofe, feet $16\frac{25}{}$ broad, how much thereof maketh a Square?

Diuide 100 by $16\frac{25}{}$ the quotient will bee $6\frac{154}{}$ almoſt: ſo much ſhall be the length of one ſquare; which multiplying the parts by 12, will be 6 feet, and inches $1\frac{848}{}$ — almoſt. And againe thoſe parts multiplied by 8, will giue ſomewhat more then 6 eight parts of an inch.

Example, III: In pauing they vſe to reckon by the yard, which is 3 feet euery way, in all 9 feet. There is a roome to bee paued, which is feet $17\frac{25}{}$ broad; how much thereof maketh a yard?

Diuide 9 by $17\frac{25}{}$, the quotient will be $0\frac{519}{}$ almoſt. the length of one yard.

6. *A fower ſided rectangle Superficies,* with all the *oppoſite ſides parallell* being propoſed, to find the *content.*

Take with your Ruler both the breadth and length of, and multiply the one number into the other.

Example, A board is feet $11\frac{17}{}$ broad, and feet $16\frac{32}{}$ long: how many feet doth it containe in all?

Multiply $16\frac{32}{}$ by $11\frac{17}{}$ the product will be feete $191\frac{}{}$ almoſt: the whole quantity of that board.

Example. II. A certaine barne tiled, hath the breadth of

of the roofe feet, 16 2/5, and the length of the barne is feet 47, how many squares of tiling hath it?

Double the length (that you may haue both sides of the roofe) and it will be 94, which being multiplied by 16 2/5, will giue feet 1527 1/5. Againe diuide those feete by 100, so shall you haue squares 15 2/7 5.

Example. III. A certaine hall paued hath the breadth feet 17 3/5, and the length feet 30 1/_, how many yards doth it containe?

Multiply 17 3/5 by 30 1/5, the product will be feet 229 1/7 5. Diuide these by 9 and the quotient will bee yards 58 17/27.

7. *A fower sided Superficies* with the *two sides of length only parallel* being proposed, to find the *content* thereof.

Take with your Ruler the length of the two parallell sides thereof: adde both those numbers together, multiply halfe that summe, by the breadth of the Superficies taken the neareſt way over, and the product will bee the content thereof.

Example. A *Trapezium*, or fower sided figure is proposed, hauing two sides thereof parallell, the length of the longer parallel side is feete 18 1/5 and the length of the shorter side is feete 14 1/5, the breadth thereof being taken the neareſt way ouer is feete 12 1/5, I would know how many feete are contained in the whole Superficies?

The length of the parallell sides are feet 18 1/5, and 14 1/5, which added together make 33 1/_, halfe whereof is 16 1/5, which multiplied by the breadth 12 1/_, the product will be 207 1/5, so many feet are contained in the superficies of the Trapezium proposed.

8. A fower ſided Superficies which hath none of the ſides paralled, as alſo euery plaine figure of more ſides the fower being propoſed, muſt with *Diagoniall* lines bee diuided into triangles. And note that euery ſuch figure containeth ſo many triangles as it hath ſides, abating two out of the number. Then thoſe triangles are to be meaſured ſeuerally as followeth.

9 To find the *content, or Area of a Triangle.*

Take the perpendicular height, or neereſt diſtance betweene the baſe or knowne ſide, and the angle oppoſite: and by that height multiply halfe the baſe, or multiply the whole baſe by halfe that perpendicular height: and the product ſhall be the content, of that Triangle.

But if it be an *Æquilater triangle*: ſay,

1000 . 433$\frac{10127}{}$ :: the ſide of the triangle. Area.

10. To find the *content of a ſegment of a Circle*, whereof the *Periferia* is giuen in *degrees and decimall parts.*

Firſt ſay, As 100000, is to 1745$\frac{32925}{}$: ſo is the Arch in degrees, to the Arch in the diuiſions of the Radius. keepe this number found.

Againe by the 6 Sect: 1 chap: find out the *true ſinus* of the Arch giuen. Then take the difference of theſe two numbers found, by ſubducting the *Sinus* out of the Arch. And laſtly multiply halfe that difference by the Radius 10000, the product ſhall be the content of that ſegment.

11. The

11 *The* chord of any arch, *together with the* Radius, *or ſemidiameter of the whole circle being giuen, to finde out the* Arch it ſelfe. *Say,*

As the Radius giuen, is to halfe the chord (reckoned in the fourth circle) *ſo is* 100000, *to the ſinus of halfe the arch* (to be reckoned in the firſt, or eighth circle). Wherfore double the arch found, and ſo haue you the arch of the chord propoſed.

12 *To finde* a Quadrat, *or* Square equall to a ſuperficies *giuen.*

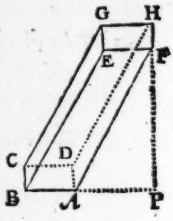
Firſt, ſeeke out (as hath beene taught) the content of that ſuperficies: then take the quadrat roote thereof by Ch. p. 5, Sect. 4.

Of Solid meaſures.

13 *In* a Columne, or Cylinder, *hauing the* baſe, *to finde* how much of it maketh a foot ſolid.

By a *Columne* I meane a ſolid body ariſing from a plaine baſe, the angular lines whereof are paralleli, and equall: and if the angular lines make right angles with the baſe, it is *a right Columne*, and the length is the height thereof: but if they make oblique angles, it is an *Oblique Columne* and the length is not the height, but the height is a perpendicular line let downe from the top of the Columne vnto the baſe, extended if neede bee: as in the Diagrame,

Diagrame, the Solid ABCDEFGH, is an *Oblique Columne*, because the angular line EB, standeth obliquely vpon the side of the base BA, and indeede vpon the base it selfe. Wherefore the *height* of it shall be equall to the line FP, let fall from the top vnto the base extended.



And after this manner also a *Cylinder*, and a *Pyramid*, and a *Cone* is esteemed either *right*, or *oblique*, and the *height* taken accordingly.

First therefore the base is to be found, of what fashion soeuer it is, as hath euen now beene shewed, either in this Chapter, or in the last before: and then diuide 1 by that same base: the quotient shall bee, the height of a Section thereof, which is equall to one foot solid.

Example. A Columne, or peece of tymber, whose sides are all parallel, hath the breadth feete 1$\frac{75}{100}$, and the thicknesse thereof is feet 1$\frac{25}{100}$: which multiplied together the product will be 2,$\frac{1875}{10000}$. Diuide therefore 1, by 2,$\frac{1875}{10000}$: and the quotient shall be 0$\frac{457143}{1000000}$ almost. And so much is the height of a solid foot, of that peece of timber.

14 *Having the* Base, *and the* height of a Columne, *or* Cylinder, *to finde the* whole content.

Multiply the base into the height, and the product shall be the content.

Example. A Columne hath the base feet 2$\frac{1875}{10000}$, and the height thereof is feet 17$\frac{14}{100}$, how many feet are contained in the whole?

Multiply

Multiply the baſe 2,1875, by 17,24 and the product will be 36,986875, ſo many ſolid feet are contained in that Columne.

And in this very manner may you finde the content of a Cylinder, hauing either the diameter, or circumference giuen, together with the height.

15 To meaſure tapering timber, *the* baſe, *or* baſes thereof, *together with the* height *being giuen.*

A Tapering peece of timber, according as the baſe thereof is right lined, or circular, is either a *Pyramide* or a *Cone*, or elſe a *ſegment* of one of theſe two: If it be a compleat Pyramide, or Cone, it hath but one baſe, Multiply that baſe by ⅓ of the height, and the product ſhall be the content.

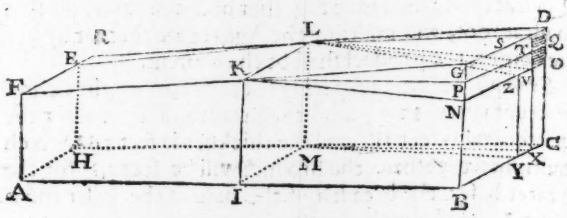
But if it be the ſegment of a Pyramide, or Cone, Firſt finde out the baſes at both the ends, and multiply the one by the other, and out of the product thereof extract the Quadrat root: then adde together both the baſes, and that quadrat root, and multiply the Aggregate thereof by ⅓ of the height, the product ſhall be the content.

Example. There is a tapering peece of timber, the height whereof is feet 12,6, and the breadth of the baſe at the greater end is feet 1,75, and the thicknes is feet 1,32, which multiplied together, the product will be feet 2,31 for the greater baſe: the breadth of the baſe at the leſſer end is feet 1,2, and the thickneſſe there, is feete 0,91, which multiplied together, the product will be feet 1,092 for the leſſer baſe. Multiply the baſes together, the product will be 2,52252, the quadrat root whereof is 1,588 almoſt, to which if you adde the ſummes of both the baſes, the aggregate will be 4,99, which being multiplied by ⅓ of the height, *ſcil.* 4,2, the product will be feet 20,958, the content of the peece of timber.

16 Wherfore that vulgar manner which *Carpenters* vſe in meaſuring of tapering timber, is not true: for if a peece of timber be tapering, they meaſure it in the very midle, and take the baſe, or Section there, multiplying it by the whole length. Which their manner of working, I ſay, is erroneous.

For firſt by practiſe a content will be giuen, euer leſſe then the true content found according to the former Sect: which way of working is infallibly true, as is *Analitically* demonſtrated, in my *Clauis mathematica*, Cha. 20, Sect. 15.

And ſecondly I ſay, that the product of that middle baſe, or Section, multiplied by the length, ſhall bee leſſe then the true content, by foure Pyramides, hauing for their baſes, a rectangle vnder halfe the difference of the thickneſſe at the ends, and a quarter of the difference of the breadths: and are as long as halfe the peece of timber: Or which is all one, by a Parallelepipedon, vnder halfe the differences of the breadthes, and thickneſſes, at both ends, and a third part of the whole length.



Which that I may ſhew, ſuppoſe one quarter of a tapering peece of timber giuen, ſawed in ſunder, at halfe the breadth, and halfe the thickneſſe be *ABCDGFEH*: the midle Section is *IKLM*. Meaſure vpon the greater baſe *BN*, and *CO*, equall to *AF*, and *BY*, and *NZ*, equall to *FE*. Diuide *CY*, and *DO* in the midſt, in the points *X*, and *Q*, and draw the lines *PQ*, *NO* parallel to *BC*, and

and *YZ*, *SX* parallel to *BG*. Meaſure alſo *FER*, equall to *GS*, or *KL*. And ſo the Parallelegram *BPTX*, ſhall be equall to the middle Section *IKLM*. Laſtly, draw the lines *LO*, *LV*, *LZ*, and *KP*, *KN*, and *MX*, *MY*.

I ſay that in this one quarter of the peece of timber, A ſolid hauing the baſes equall to *IKLM*, and the length *AB* (which is the vſuall meaſuring of Carpenters) is leſſe then the true content, by the Pyramide *DSVOL*, in which *DO*, or *SV*, is halfe the difference of thicknes, and *DS*, or *VO*, one quarter of the difference of breadths in both ends: and the height of it equall to halfe the length.

For the two ſolides *AFEHIKLM*, *BNZYIKLM*, are apparantly equall, againe the two ſolide wedges *GSTPKL*, *PTVNKL* are equall, and alſo the two ſolide wedges *OCXVLM*, *VXYZLM* are equall. Now if you turne ouer the wedge *GSTPKL*, vnto the leſſer part *AFEHIKLM*, it will ouer-reach in thickneſſe, at the leſſer end the quantity of *ER*, and if you alſo turne ouer the wedge *OCXVLM*, vnto the leſſer part, you ſhall find it to fill vp the former ouer-reaching, and to make an exact Parallelepipedon, the baſes or ends whereof are equall to the middle Section *IKLM*. but ouer and aboue thoſe two wedges turned ouer, you ſhall haue left the Pyramide *DSVOL*, which was to be proued.

And in like manner if a round tapering ſolide, or peece of timber, be meaſured by the middle Section, or circular baſe, I ſay, That the product thereof multiplied by the length, ſhall be leſſe then the true content, by a Cylinder, the diameter of whoſe baſe, is equall to halfe the difference of the diameters of the two baſes: and the length is one third part of the whole length.

CHAP. IX.

Concerning the Meaſuring, *or* Gauging of Veſſells.

1 

A Wine, or Beere veſſell, whether Pipe, Hogſhead, Barrell, Kilderkin, or Firkin, and ſuch like, is in forme of a *Spheroides*, hauing the two ends equally cut off: and accordingly may be meaſured thus.

Meaſure the two diameters of the Veſſell, in inches, or elſe in tenth parts of a foote, the one at the bung hole, the other at the head, and alſo the length within. And by the diameters found, finde out the circles; then adde together two third parts, of the greater circle, and one third part of the leſſe: Laſtly, multiply the aggregate by the length: ſo ſhall you haue the content of the Veſſell, either in Cubic inches, or cubic tenth parts of a foot.

Example. Suppoſe a veſſell, having the Diameter at the bung 32 inches, and at the head 18 inches, and the length 40 inches. The quadrat of 32 is 1024, and the quadrat of 18 is 324: Say then euermore,

1 . 0,$\frac{12}{}$36 :: 1024 . 536,$\frac{16}{}$6, $\frac{2}{3}$ the circle at the bung.

1 . 0,$\frac{26}{}$18 :: 324 . 84,$\frac{82}{}$3, $\frac{1}{3}$ circle at the head.

Or elſe by Chap. 7, Sect. 4.

The aggregate of thoſe two circles is 620,$\frac{98}{}$9, which being multiplied by 40, the length giueth 24839,$\frac{56}{}$ cubic inches for the whole content of that veſſell.

M. Edw.

2 Mr. *Edm: Gunter* in his ſecond booke of the Croſſe Staff, *Chap.* 4. pretending to ſhew the manner of gauging Wine veſſels, beginneth with theſe words.

" *The Veſſels which are heere meaſured, are ſuppoſed to bee* Cylin-
" *ders, or reduced into* Cylinders *by taking the meane, betweene the Di-*
" *ameter at the head, and the Diameter at the boungue, after the uſuall*
" *manner.*

And according to this ſuppoſition, teacheth to finde a *Gauge point*, for a gallon of wine, in that his imagined *Cylindriacall veſſell.*

Becauſe his words are cautelous, and ſubterfugious, wee muſt a little examine them; for if his way bee true, my Rule before ſet downe, though grounded vpon demonſtration, cannot ſtand.

Well then, that reduction of a wine Veſſell into a Cylinder, is either true, or falſe; if it bee true what needs thoſe ambiguities of *Veſſels which are here meaſured:* and *are ſuppoſed to be, &c.* and *after the vſuall manner?* if falſe why is it not noted, but deliuered as a Rule to confirme an error. And what meaneth, *the meane betweene the Diameter at the head, and the Diameter at the boungue?* is it the *meane Geometricall*, or *Arithmeticall*, that is the *meane proportionall*, or *that which equally differeth* from both? ſuch ſhifting is vnworthy an Artiſt.

Firſt therefore, Let it be *the meane in reſpect of difference*, which is equall to *halfe the ſumme of the two Diameters:* I ſay that the Veſſell cannot truely be reduced to a *Cylinder* by ſuch *a meane Diameter.* For ſeeing it is moſt apparent that ſuch a Veſſell, is greater in the middle then at the ends, the boords, or ſides thereof, ſhall from the middle to the ends, goe either ſtreight, and ſo the Veſſel ſhall be as it were, two equall ſegments of Cones, ſet baſe to baſe: Or elſe arching, and ſo the Veſſel ſhall (as before I ſayd, and is commonly taken for a truth) bee a *Spheroides*, hauing the two ends equally cut of.

If it be conſidered as two ſegments of Cones: the meaſure by that *meane Diameter*, or midle Section is quite falſe, as hath beene demonſtrated in the former Chapter, Sect: 16, 17. and will be giuen leſſe then the true content, although the ſides goe ſtraight: much more then, if the ſides goe arching; for that conuexneſſe, muſt needs yeeld a greater capacity. And therefore in neither can that manner of Gauging be true.

Againe, if the *meane Diameter* be vnderſtood to be the *meane proportionall betweene the two Diameters*, it is much more falſe, for betweene any two numbers, the *meane Geometricall* is leſſe then the *meane Arithmeticall*.

Thus much I haue thought good in this place ingenuouſly to ſignifie to the inexpert Learner, that hee might not beguile himſelfe with a prejudged opinion.

3. I haue ſhewed *the meaſuring of veſſels by the cubic inch*: but our vſuall reckoning is by the *Gallon*, and parts thereof. Wee muſt therefore doe the beſt we can, to inquire the *true quantitie of a Gallon in inch meaſure*, which will bee difficult to doe exactly, both becauſe the *Standards* vſually are not ſtreight ſided, but a little arching, neither doe they agree perfectly one with another: but what partly by experience, both mine owne, and others, which hath come to my ſight; and partly by reaſoning ſhall ſeeme to me moſt probable, I will not refuſe to ſet downe.

4. Our *Engliſh Gallon* is vnderſtood to bee either in *Ale meaſure*, or *Wine meaſure*: and theſe two meaſures not a little differing. And firſt we will inquire about our *Ale meaſure*.

I my ſelfe haue meaſured Buſhels, and Pecks, which haue exactly beene fitted to the *Standards*, and haue ſtill in my account found a *Gallon to containe better then* 270 *cubic*

cubic inches, indeed much about 272, or 273 as preciſely as I could meaſure in a veſſell not truely regular. Alſo my worthy friend *Maſter William Twine*, who hath vndergone great paines and charge, in finding out the *true content of our Engliſh meaſures*, gaue vnto me two ſeuerall meaſures of an *Ale-gallon*, and thoſe in due conſideration but little differing. The one was found out by a braſſen veſſell made in manner of a Parallelepipedon, the baſe whereof was exactly ſixe inches ſquare, and the ſides diuided into inches and twentieth parts: into which veſſell he powring out a *ſtandard Gallon of Queene Elizabeth*, filled with water, found it therein to attie vnto 7 inches, and 6 tenth parts: which being computed maketh cubic inches 273$\frac{6}{10}$. The other was found by taking the dimenſions of that *ſtandard Gallon*, which was made in forme of *a ſegment of a Cone*, but that the ſides were a little arching: the dimenſions were thus; the Diameter of the top was inches 6$\frac{5}{10}$: the Diameter of the bottome was inches 5$\frac{13}{10}$: and the height of it was inches 9$\frac{8}{10}$: which being caſt vp by Chap: 8, Sect: 15, will be found to containe cubic inches 268$\frac{185}{1000}$: differing from the former, only cubic inches 4$\frac{3}{4}$: which difference might well ariſe through the curuitie of the ſides. Theſe meaſures he did not only take himſelfe, but to giue me ſatisfaction, ſhewed me the experience in the ſaid *Veſſell and Standard*: but the truth is, I obſerued the *Standard*, beſides the arching of the ſides, to bee not exactly circular within, nor the brimme of an euen height, nor the bottome plaine: and in taking the height of the water in the *Veſſell*, our ſight was not able to eſtimate the aſcent thereof ſo preciſely, that a ſpoonefull of water, more or leſſe, could breed any ſenſible difference.

What therefore ſhall wee doe in this difficultie? indeede looke to the firſt ground, and principle of our Engliſh meaſuring, from Barley cornes. For the length of

3 Barley

3 Barley cornes taken out of the middle of the eare is an *Inch*, or *Uncia*, that is a twelfth part of a foot. 3 feete make a *Yard*: and 16 feet and an halfe, that is 5 yards and an halfe, a *Perch*, with which wee meaſure our land; for 40 perches is a *Furlang*, and 8 furlongs an *Engliſh mile*: and againe 40 ſquare perches is a *Roodland*, 4 roodland make an *Acre*. So then a Perch which is feet $16\frac{1}{2}$, or yards $5\frac{1}{2}$ is as it were the beginning of all land meaſure in length: and a ſquare Perch which is feete $272\frac{1}{4}$, is as it were the beginning of all land meaſure in the ſuperficiall content.

Now therefore ſeeing in *Veſſels a gallon is as it were the beginning of Veſſell meaſure* (for a *Pottell* is but a diminutiue of it, and *a quart*, the quarter) it is not vnlikely that our wiſe Anceſtors had ſuch a conſideration alſo in ſolide meaſures, that as a ſquare Perch (the beginning of Superficiall land meaſure) did containe 272 ſquare feete and a quarter; ſo a Gallon (the beginning of veſſell meaſure) ſhould containe 272 Cubic inches and a quarter. And the rather ſeeing that the ancient *Geographers*, diuide a foote into 4 *Palmes*, a palme being 3 of our inches, as 3 feete are a yard. So that as the ſide of a ſquare Perch conſiſteth of yards $5\frac{1}{2}$, a Gallon alſo ſhould conſiſt, of a number of Cubic inches the ſquare ſide whereof is palmes $5\frac{1}{2}$.

Wherefore ſauing the exact truth when it ſhall appeare, and in the meane time the more probable reaſons of other men, I make bold to tender this my coniecture, to the cenſures of more diligent Inquirers, *That the meaſure of an Engliſh Ale gallon ſhould be a ſquare Veſſell of inch* $16\frac{1}{2}$, *or Palmes* $5\frac{1}{2}$ *every way, and* 1 *inch deepe*: that is $272\frac{25}{100}$ Cubic inches.

5. And this my opinion may peraduenture receiue ſome confirmation by the inquiry of an *Engliſh Wine Gallon*.

Maſter

M. *Henry Briggs* that learned Geometrician, and my very louing friend, made an experiment, with a Cubicall veſſell, which was 12 inches euery way, which hauing filled with water carefully meaſured, found it to containe 7 gallons and an halfe wanting *a moment*, as hee himſelfe long ſince, being then of *Greſham Colledge*, ſignified to me. Now if it had contained exactly 7 gallons, and an halfe, a *Wine gallon* ſhould haue beene 230¼ cubic inches, but becauſe it wanted a little, the gallon muſt be ſomewhat bigger, for which *moment* therefore if you will put 6 hundred parts of an inch, the *Wine gallon ſhall containe* 231 *cubic inches.*

Againe M. *Gunter* in the place before mentioned ſheweth, that the common opinion is that at *London*, a *Cylindriacall veſſell*, whoſe diameter is 38 inches, and length 66 inches, doth containe 324 gallons: wherefore by this account a gallon ſhould bee 231 cubic inches almoſt exactly, which in both ſo neerely agreeing, wee may well conclude, *That an Engliſh Wine gallon doth containe* 231 *cubic inches.*

It is alſo commonly receiued, that the reaſon of the greatneſſe of an *Ale gallon* aboue the *Wine gallon* is, that becauſe of the frothing of the Ale or Beere, the quantity becommeth leſſe, and therefore ſuch liquors that did not ſo yeeld froth, as Wine, Oyle, and the like, ſhould in reaſon haue a leſſer meaſure. If then we compare theſe two gallons together, we ſhall finde that

272,25 . 231 :: 161⅝ . 14 .

which abatement might to our Anceſtors, in apportioning thoſe meaſures, ſeeme to be reaſonable.

6 *To finde* how many Cubic tenth parts of a foot are in a gallon, *both of* Beere, *and* Wine: *or also in* any number of Cubic inches.

Because there be in a Cubic foote, 1000 Cubic tenth parts, and 1728 Cubic inches, say,

1728 (C: 12) . 1000 (C: 10) :: $272\frac{1}{25}$. $157\frac{5}{5521}$ — cubic tenth parts of a Beere gallon.

And 1728 (C:12) . 1000 (C: 10) :: 231 . $133\frac{68}{803}$ — cubic tenth parts in a Wine gallon.

Also 1728 . 1000 :: $24839\frac{56}{}$. $14374\frac{746}{}$, cubic tenth parts, are in the vessell measured in Sect. 1.

7 And according to these measures so found, you may easily finde the content of a *pint*, or *quart*, or *peck*, or *bushell*, which two last are to be reckoned in *Ale measure*.

8 *The* Content of a Vessell, *being giuen* in cubic inches, *or in* cubic tenth parts of a foot, *to finde* how many gallons it containeth.

This is easily done if you diuide the content giuen in inches, by $272\frac{1}{25}$ for *Ale measure*: and by 231, for *Wine measure*. But if the content be giuen in decimall parts of a foote diuide it by $157\frac{5}{5521}$ — for Ale measure; and by $133\frac{68}{803}$ — for Wine measure.

Example. How many wine gallons are in a vessell containing $24839\frac{56}{}$ cubic inches, or $14374\frac{746}{}$ cubic tenth parts of a foote. Diuide $24839\frac{56}{}$ by 231, or diuide $14374\frac{746}{}$ by $133\frac{68}{}$, and the quotient shall be $107\frac{53}{}$ wine gallons.

CHAP.

CHAP. X.

Concerning the Comparison *of ſundry* Metals, *in* quantity *and* weight.

1 IF foure pieces of Metals, whereof the third is of the ſame kinde with the firſt, and the fourth of the ſame kinde with the ſecond, are proportionall, their gravities alſo, or weights, ſhall be proportionall.

2. If there be foure pieces of Metall, whereof the third is of the ſame kinde with the firſt, and the fourth of the ſame kinde with the ſecond: and the firſt and ſecond be of equall greatneſſe, and the third and fourth of equall weight; the weights of the firſt and ſecond, ſhall be reciprocall to the magnitudes of the third and fourth.

3. Two Sphæres of the ſame matter are in weight, as the cubes of their Diameters, are in magnitude. *Et contra.*

4. Pieces of Metall, if they be of equall magnitude, have their weights in direct proportion, as is here ſet downe: but if they be of equall weight, they have their magnitudes in proportion reciprocall: According to the experiments of *Marinus Ghetaldi*, in his Tractate called *Archimedes promotus.*

Gold.	3990	Braſſe.	1890
Arg. Viu.	2850	Iron.	1680
Lead.	2415	Tinne.	1554
Silver.	2030		

5. To finde the *Weight of a Sphære of Tinne*, having the *Diameter* 1 *Inch*, *or elſe* 1 *tenth part of a foot.*

Take a piece of Tinne, and turne it exactly in a Lathe, into a Cylinder, having both the Diameters of its baſe, and alſo the length equall, to the Diameter of the Sphære given. Then weigh that Cylinder, that you may have

the weight thereof in graines: And laſtly, take two third parts of the whole number of graines: for the weight of the Sphære.

After this manner *Marinus Ghetaldi* found a Cylinder of 1 inch, or twelfth part of a foot thick and long, to weigh 1824 graines: whereof $\frac{2}{3}$ is 1216, the weight of a Sphære of that thickneſſe.

Againe if you ſay,

1000 (C: 10) . 1728 (C: 12.) : : 1216. $2101\frac{248}{1000}$

You ſhall have the weight of a Sphære whoſe Diameter is one tenth part of a foot.

Wherefore alſo *a Cubed inch of Tinne weigheth* $2322\frac{4}{10}$, and *a Cubed tenth part of a foote weigheth* $4013\frac{1}{10}$.

And note that *Mar: Ghetaldi* uſeth the ancient *Roman foot*, which by the meaſure ſet downe in his booke ſeemeth to be very little leſſe, then our uſuall *Engliſh foot*, if not exactly the ſame.

Note alſo that he divideth *one pound*, into 12 *ounces*; and every *ounce* into 24 *Scruples*, and every *Scruple* into 24 *graines*: So that *an ounce* with him weigheth 576 *graines*: and *a pound* 6912. Whereas our *Engliſh pound of Troy weight by Aſſize*, or *Gold-ſmiths weight*, is but 5760 *graines*, and our *ounce* 480. But whether the *Raguzean graine*, be the ſame with our *Engliſh*, I leave to be tryed by the diligent Practizer.

6 To finde the *Weight of a Sphære of Tinne, at any other Diameter aſſigned.*

Multiply the Cube of the Diameter given by 1216, if it be in inch meaſure; or by $2101\frac{248}{1000}$, if the meaſure be by decimall parts of a foot: and the product will be the weight of that Sphære.

And contrariwiſe to finde the Diameter of a Sphære of Tinne, by the weight given in graines. Divide the weight given in graines by 1216 if you would have inch meaſure:

meaſure: or by 2101,^{248} if you would meaſure by decimall parts of a foot, and the quotient ſhall be the Cube of the Diameter.

7. To finde the *Weight of a Sphære of any Metall, at any Diameter* given, either *in inch meaſure*, or *in decimall parts of a foot.*

Firſt by ſect. 6, ſeeke the weight of a Sphære of Tinne, at that Diameter: then by Sect. 4. ſay, *As the proportionall number of Tinne, is to the proportionall number of that other Metall: ſo is the weight of the Sphære of Tinne now found, to the weight of the Sphære propoſed.*

Example. Suppoſe a Sphære of Iron, whoſe diameter is 3 inches: what ſhall be the weight thereof?

Firſt, the weight of a Sphære of Tinne, of 3 inches Diameter, will be found to be 32832 graines. Then ſay,

1554 . 1680 : : 32832 . 35494,^{054} graines, the weight of the Sphære propoſed.

8 To finde the *Diameter of a Sphære of any Metall in inch meaſure, or decimall part of a foot,* the *weight* thereof being given.

Firſt by the contrary of Sect. 6, ſeeke the Cube of the Diameter of a Sphære of Tinne of that weight. Then by Sect. 4. ſay reciprocally. *As the proportionall number of that other Metall, is to the proportionall number of Tinne: ſo is the Cube of the Diameter now found, to the Cube of the Diameter of the Sphære propoſed.*

Example. A Sphære of Iron weigheth 35494,^{054} grains: how many inches is the Diameter thereof?

Firſt, the Cube of the Diameter of a Sphære of Tinne of 35494,^{054} graines weight, will be 29,^{18910695}, then ſay, reciprocally,

1680 . 1554 : : 29,^{18910695} . 27—

The Cubic root whereof is 3, the Diameter of a Sphære of Iron of that weight propoſed.

CHAP. XI.

Concerning the *Ordering of Souldiers*, *in any kinde of rectangular forme of battaile.*

1 BAttailes are considered either in respect of the number of men, or in respect of the forme of ground, *A square battaile of men* is that which hath an equall number of men, both in *Rank and File*, though the ground on which they stand, be longer on the *File*, then on the *Ranke*. And *a square battaile of ground* is that which hath the *Ranke* as long as the *File*, though the men in *Ranke* be more then in *File*.

2. In respect of the number of men, it is called either *a square battaile*, or *a double battaile*, or *a battaile of the grand front*, which is quadruple, or *a battaile of any proportion, of the number in Ranke, to the number in File.*

3. If it be *a square battaile of men*: Extract the quadrat root out of the whole number of men, and the same shall be the number of Souldiers, to be set in a *Ranke*.

Example. 576 Souldiers are to be martialled in a square battaile, that so many may be in *Ranke*, as in *File*.

Take the quadrat root of 576, which is 24: the same shall be the number to be placed in a *Ranke*.

4. If it be *a double battaile of men*: Extract the quadrat root out of halfe the number of men, and the same doubled shall be the number of Souldiers to be set in a *Ranke*.

Example. 1458 Souldiers, are to be placed in a double battaile; so that twise so many may be in *Rank* as in *File*.

Take

Take halfe the given number 1458, which is 729, the quadrat root whereof is 27: double it and you ſhall haue 54 men to be placed in a ranke.

5. If it be *a quadruple battaile*, which is called *of the great front*: Extract the quadrat root out of one quarter of the number of men, and the ſame quadrupled ſhall be the number of Souldiers to be ſet in a ranke.

Example. 1024 Souldiers are to bee martialled into a *battaile of the grand front*, ſo that fower times ſo many be in *ranke* as in *file*.

Take one quarter of 1024 the number given, which is 256, the quadrat root whereof is 16: quadruple it, and you ſhall haue 64 men to be placed in a ranke.

6. If a battaile bee required of any other forme, that is, if a Ratio be given, according to which the number of men in Ranke, ſhall be to the number in file. Multiply the two termes of the Ratio given: Then ſay *As the product is to the quadrat of the terme which is for the ranke*, or *As the terme which is for the file, is to the terme which is for the ranke: ſo is the whole number of Souldiers, to the quadrat of the number of men to be placed in a ranke.*

Example. 1944 Souldiers, are to be martialled ſo, that the number of the ranke, be to the number of the file, as 8 vnto 3, that is for 8 men in ranke, 3 are to be ſet in file.

Firſt multiply the two termes of the Ratio 8 and 3, the product whereof is 24, alſo quadrat 8, the terme of the ranke, which will be 64. Then ſay,

3 . 8 :: 1944 . 5184 .

out of which extract the quadrat root 72, and it will giue you the true number of the ranke.

7. In reſpect of the forme of ground, the battaile is either

either *a ſquare of ground*, or longer one way then the other. For the diſtance, or order of Souldiers martialled in array, is diſtinguiſhed either into *Open order*, or *Order*.

Open order is when the very centers of their places, are diſtant 7 feet aſunder, both in ranke and file.

Order is when the centers of their places, are diſtant 3 feet and a halfe in *ranke*, and ſo much in *file*. Or elſe 3 feet and a halfe in *ranke*, and 7 feet in *file*: which laſt *order*, and whatſoeuer *order* elſe there is, in which the diſtance of the *rankes* one from another is greater, then the diſtance of *files*, cauſeth that *a ſquare of men*, maketh not *a ſquare of ground*, but the ground is longer on the *file* then on the *ranke*.

8 If it be *a ſquare battaile of ground*, the centers of the diſtances being feet $3\frac{1}{2}$ in *ranke*, and 7 feet in *file*. Becauſe $3\frac{1}{2}$ is halfe of 7, the ratio of the diſtances, is as 1 unto 2. And ſeeing the number in ranke, to the number in file, is reciprocall to the diſtances, the ratio of the number of men in ranke, to the number of men in file, ſhall be as 2 unto 1. And ſo the Rule ſhall be the ſame with that in Sect. 6, namely, *As the terme of the file, is to the terme of the ranke; ſo is the whole number of Souldiers, to the true number of the ranke.* the quadrat of

Example, 1352 Souldiers, are to bee ſet in *a ſquare of ground*, that their diſtances may be feet $3\frac{1}{2}$ in ranke, and 7 feet in file.

The Ratio of the *ranke* to the *file*, ſhall reciprocally be, as 7 to $3\frac{1}{2}$, that is as 2 to 1. Say therefore

1 . 2 :: 1352 . 2704,

the quadrat roote whereof 52 is the number of men to be ſet in a ranke.

9 If a battaile wherein the diſtance in ranke is vnequall to that in file, be longer one way then the other, accor-

ding

ding to any Ratio giuen: there is to be considered a double ratio, one reciprocall in respect of the distances, the other according to the forme of the ground. Wherefore to finde the Ratio of men in rank, to the men in file, Multiply the two termes of the ranke, for the ranke, and the two termes of the file, for the file. And then the Rule shall bee the same with that in Sect. 6, namely, *As the terme of the file, is to the terme of the ranke: so is the number of Souldiers, to the quadrat of the true number of the ranke.*

Example. 10290 souldiers, are to be set in a battaile, so that they may stand onely 3 feet asunder in ranke, and 7 feet in file, and the length of the ground for the ranke, to the length of the ground for the file, shall haue the ratio of 5 vnto 2.

First in respect of the distances, the Ratio of Rank to file, reciprocally is as 7 vnto 3. Secondly, in respect of the ground, the ratio of rank, to file, is as 5 to 2. Wherefore by multiplication of like termes, the true ratio of rank to file shall be 7*5 to 3*2, that is as 35 to 6. Say therefore

6 . 35 :: 10290 . 60025

the quadrat root whereof is 245, the number of men to be set in ranke.

10 If 1000 Souldiers, may be lodged in a square, of 300 feete, how many feete must the side of a square be, which will serue to lodge 5000? Say,

1000 . 5000 :: 300*300 . 450000,

the quadrat roote whereof 671 — is the square side sought for.

And this is the order for resolution of all other questions of this sort.

CHAP. XII.

A collection of the most necessarie Astronomicall operations,

BEfore wee deliuer the Rules of such operations, it will not be inconuenient, to set downe certaine *Reductions*, whereof we may haue frequent vse.

To reduce sexagesime parts into decimals. Diuide the sexagesimes giuen by 60.

Example. How many decimals are 34′, 12″?

Here are required two reductions, first of the seconds into decimals of minutes: then of the minutes with their decimals, into decimals of degrees, Thus

60 . 1 :: 12″ . $0\frac{12}{}$

Againe

60 . 1 :: $34\frac{12}{}$′ . $0\frac{57}{}$°

Wherefore 34′, 12″ are equall to $0\frac{57}{}$ of a degree.

And contrariwise *to reduce decimall parts of degrees in sexagesimes.* Multiply the decimall part giuen by 60.

Example. How many sexagesime parts are $0\frac{57}{}$°?

1 . 60 :: $0\frac{57}{}$° . $34\frac{12}{}$′

Againe

1 . 60 :: $0\frac{12}{}$. 12″

To *reduce houres into degrees.* Multiply the houres with their decimall parts by 15.

Example. How many degrees are $8^{Ho}, 34', 12''$; that is by the former reduction $8\frac{57}{100}{}^{Ho}$? thus

1 . 15 :: $8\frac{57}{100}$. $128\frac{55}{100}$

Wherefore Houres 8, 34', 12'' doe containe $128\frac{55}{100}$ degrees.

And contrariwiſe *to reduce degrees into houres.* Diuide the degrees with their decimall parts by 15.

Example. How many houres are in degrees $128\frac{55}{100}$?

15 . 1 :: $128\frac{55}{100}$. $8\frac{57}{100}$

2 It is to be vnderſtood, that if foure numbers are proportionall, their *Order* may be ſo tranſpoſed, that each of thoſe termes, may bee the laſt in proportion. In this manner,

I. As the firſt is to the ſecond; ſo is the third to the fourth.

II. As the third is to the fourth; ſo is the firſt to the ſecond.

III. As the ſecond is to the firſt; ſo is the fourth to the third.

IIII. As the fourth is to the third; ſo is the ſecond to the firſt.

Wherfore euery proportion doth implicitly containe foure *Orders*, two deſcending, and two aſcending, as may be ſeene by their combinations: By one of which *orders*, if of foure proportionall numbers, any three be giuen, that other which is vnknowne, may be found out.

Example. To finde out any of theſe,

termes
1 As the Sine of the complement of the *ſuns declination*,
2 is to the Sine of the compl. of his *altitude*;
3 So is the Sine of the *Sunnes Azumith* from the meridian,
4 to the Sine of *the horary diſtance from the meridian.*

If the firſt, ſecond, and third termes be giuen, the fourth ſhall be found out by the *I order.*

If the firſt, third, and fourth termes be giuen, the ſecond ſhall be found out by the *II order.*

If the firſt, ſecond, and fourth termes bee giuen, the third ſhall be found out by the *III order.*

If the ſecond, third, and fourth termes be giuen, the firſt ſhall be found out by the *IIII order.*

3. To finde out any one of theſe.

termes
1 As the *Radius*, or totall Sine
2 is to the Sine of the *diſtance, or longitude of the Sunne* in the Ecliptic, from the next Æquinoctial point:
3 So is the Sine of the *Sunnes greateſt declination* (which is the angle of the Ecliptic with the Æquinoctial),
4 To the Sine of the *Sunnes declination* in that longitude.

4. To

4. To finde out any one of theſe.

termes
1 As the *Radius*,
2 is to the Sine of the *Sunnes right aſcenſion*, from the next æquinoctiall point:
3 So is the tangent of the *Sunnes greateſt declination*,
4 to the tangent of the *Sunnes declination* in that place.

5. To finde out any one of theſe.

termes
1 As the *Radius*
2 is to the Sine of the compl. of the *Sunnes greateſt declination*:
3 So is the tangent of the *longitude of the Sunne* from the next æquinoctiall point.
4 to the tangent of the *Right aſcenſion of the Sunne*, from the ſame æquinoctial point.

6. To finde out any one of theſe.

termes
1 As the *Radius*
2 is to the Sine of the compl. of the *longitude of the Sunne* from the next æquinoctiall point;
3 So is the tangent of the *Sunnes greateſt declination*,
4 to the tangent of the compl. of the *angle of the Ecliptic with the Meridian*.

7. To finde out any one of theſe.

termes
1 As the *Radius*,
2 is to the Sine of the *Sunnes greateſt declination*:
3 So is the Sine of the compl. *of the Sunnes right aſcenſion* from the next æquinoctial point,
4 to the Sine of the compl. of the *Angle of the Ecliptic with the Meridian.*

8. To finde out any one of theſe.

termes
1 As the Sine of the compl. of the *Poles height*,
2 is to the *Radius*;
3 So is the Sine of the *Sunnes declination*,
4 to the Sine of the *Sunnes Amplitude ortiua*, that is the arch of the horizon from the place of the Sunnes riſing or ſetting to the true Eaſt, or Weſt point.

9. To find out any one of theſe.

termes
1 As the *Radius*,
2 is to the Sine of the *Sunnes greateſt amplitude ortiua*, which is in the Tropicks:
3 So is the Sine of the *longitude of the Sunne* from the next æquinoctial point,
4 to the Sine of the *Sunnes amplitude ortiua.*

Or

Or alſo of theſe.

termes
1 As the Sine of the compl. of the *Poles height*,
2 is to the Sine of the compl. of the *Sunnes greateſt declination*:
3 So is the Sine of the *Sunnes longitude* from the next æquinoctial point,
4 to the Sine of the *Sunnes amplitude ortiue*.

10. To finde out any one of theſe.

termes
1 As the *Radius*,
2 is to the tangent of the *height of the Pole*.
3 So is the tangent of the *Sunnes declination*,
4 to the Sine of the *Sunnes Aſcenſional difference*.

11. To finde out any one of theſe.

termes
1 As the *Radius*
2 is to the Sine of the *height of the Pole*:
3 So is the tangent of the *Sunnes amplitude ortiue*,
4 to the tangent of the *Sunnes aſcenſionall difference*,

12 To

12. To finde out any one of theſe.

terms
1 As the Sine of the compl. of the *Sunnes declination,*
2 is to the *Radius:*
3 So is the Sine of the compl. of the *Sunnes amplitude ortiue,*
4 to the Sine of the compl. of the *Sunnes aſcenſionall difference.*

13. To finde out any one of theſe.

terms
1 As the tangent of the *height of the pole,*
2 is to the *Radius:*
3 So is the tangent of the *Sunnes declination,*
4 to the Sine of the *Suns horary diſtance from the Meridian, being due Eaſt or Weſt.*

14. To finde out any one of theſe.

terms
1 As the Sine of the *height of the pole*
2 is to the *Radius:*
3 So is the Sine of the *Sunnes declination,*
4 to the Sine of the *Sunnes altitude being due Eaſt, or Weſt.*

15. To

15. To find out any one of theſe.

termes
1. As the *Radius*,
2 is to the Sine of the *height of the Pole*:
3 So is the Sine of the *Sunnes declination*,
4 to the Sine of the *Sunnes altitude aboue the Horizon at ſixe of the Clocke.*

16. To find out any one of theſe.

termes
1 As the *Radius*,
2 is to the Sine of the complement of the *Poles height.*
3 So is the tangent of the *Sunnes declinatiᵒn.*
4 to the tangent of the *Sunnes Azumith frᵒm the North meridian, at 6 of the Clocke.*

17 The *hower of the Sunnes Riſing, and ſetting* is found out by the *Aſcenſionall difference.* For if you reduce the degrees of the Aſcenſionall difference, into howers, it will ſhew you how much the Sunne riſeth, or ſetteth before, or after 6 a Clock.

18. The *Oblique aſcenſion* alſo of the Sunne is found out by the *Aſcenſionall difference.* For if you ſubduct the Sunnes Aſcenſionall difference, out of the right aſcenſion of the Sunne, from the beginning of *Aries*, for the ſixe Northerne ſignes which are ♈, ♉, ♊, ♋, ♌, ♍; or if you adde it thereto, for the ſixe Southerne Signes, which are ♎, ♏, ♐, ♑, ♒, ♓, you ſhall have the Sunnes oblique aſcenſion.

19. The *declination of the Sunne*, and his *Altitude* aboue the Horizon at any time, together with the *height of the Pole* being giuen, to find the *hower of the day.* Say,

As the *Radius*,
is to the Sine of the complement of the *Sunnes declination.*
So is the Sine of the compl. of the *Poles height*,
to a fourth number. Keepe it,

Then out of the Sunnes diſtance from the North pole, ſubduct the complement of the Pole; and of that remaines, and the complement of the Sunnes altitude, take both the *Summe*, and alſo *the difference.* And ſay againe,

As the *fourth before kept*,
is to the Sine of *halfe that ſumme*:
ſo is the Sine of *halfe the difference*,
vnto a number which being multiplied
by the Radius, is equall to the quadrat,
of the Sine of halfe the Angle of the
Sunnes horary diſtance from the Meridian.

20. The *declination of the Sunne*, and his *altitude* aboue the Horizon at any time together with the *height of the Pole* being given, to find the *Sunnes Azumith.* Say,

As the *Radius*
is to the Sine of the compl. of the *Suns altitude.*
So the Sine of the compl. of the *Poles height*,
is to *a fourth*, Keepe it,

Then out of the complement of the Sunnes altitude, ſubduct the complement of the Pole; and of that remaines, and the diſtance of the Sunne from the North Pole, take both the *Sunne*, and alſo the *difference*: and ſay againe,

As

As the *fourth before kept,*
is to the Sine of *halfe the ſumme:*
So is the Sine of *halfe the difference,*
vnto *a number* which being multiplied
by the Radius, is equall to the quadrat of
the Sine of halfe the angle of the *Sunnes*
horizontall diſtance from the Meridian.

21. To find the *length of the Crepuſculum,* or *Twilight.*

Betweene the light of the day, and the darkeneſſe of the night, the Twilight is ſet by the wiſe Creator; that wee here vpon the earth might not in an inſtant, paſſe from one extreame into another, but by ſucceſſiue degrees. *The Twilight is nothing elſe but the refraction of the Sunnes beames, in the denſitie of the aire.* And *Pet. Nonnius* to find the length of the Twilight, watched the time after Sunne ſet, when the twilight in the Weſt was ſhut in, ſo that no more light appeared there, then in any other part of the ſky neere the Horizon: then by one of the knowen fixed Stars, having taken the true hower of the night, found by many obſeruations, that at the time of ſhutting in the Twilight, the Sunne was vnder the Horizon 18 degrees, and vntill the Sunne was gone ſo low, the Twilight continued. Say therefore

As the *Radius*
is to the Sine of the compl. of the *Sunnes*
declination:
So the Sine of the compl. of the *heigth*
of the Pole,
is to a fourth, Keepe it.

Then out of the Sunnes diſtance from the South Pole, ſubduct the complement of the Pole; and of that re-

maines, and degrees 62, take both the *Summe* and alſo the *difference*; and ſay againe,

As the *fourth kept*,
is to the Sine of *halfe the ſumme*:
ſo is the Sine of *halfe the difference*,
to *a number* which being multiplied
by the Radius is equall to the quadrat
of the Sine of *halfe the angle of the*
Sunnes diſtance at the ending of the Twi-
light, from the high Noone next to it.

Wherefore if out of the whole angle conuerted into howers, you ſubduct halfe the diurnall arch, or the hower of the Sunnes ſetting, you ſhall haue the true length of the *Crepuſculum*, or *Twilight*.

22. To find the *length of the leaſt Crepuſculum in the yeare.*

The Sunne being in the winter *Tropic* maketh the longeſt Crepuſculum, of the whole winter halfe yeare, and from thence, as the dayes increaſe, the *Crepuſcula* doe decreaſe vntill they come to bee ſhorteſt, which is in a certaine Parallel, betweene that Tropic, and the Æquinoctiall: the declination whereof is thus found out.

As the tangent of the *complement of the Pole*,
is to the *Sine thereof*:
So is the *tangent of* 9 *degrees*,
to the Sine of the *declination of the Parallell,*
in which the Sunne maketh the ſhorteſt Cre-
puſculum of the whole yeare.

23 But

23 But before the *Crepuſculum* come to bee ſhorteſt, there is another Parallel, in which the Crepuſculum is equall to that in the Æquinoctiall: the declination wherof is thus found out.

As the *Radius*
is to the Sine of the *altitude of the Pole*:
So is the *Sine of* 18 *degrees*
to the Sine *of declination of the Parallel in which the Sunne maketh the Twilight equall to that in the Æquinoctiall.*

24. *If an Arch of the Ecliptic, be equall to his Right aſcenſion, one end thereof* beeing knowne, to find out the *other end.* Say,

As the Sine of the Compl. *of the declination of the arch ginen.*
is to the *Radius*:
ſo is the Sine of the compl. of the *greateſt declination,*
to the Sine of the compl. of the *other end.*

25. To find the *poynt of one quadrant of the Ecliptic,* wherein the *difference of longitudes ceaſe to be greater, then the differences of the right aſcenſions.*

Multiply the Sine of the complement of the greateſt declination, by the Radius, and out of the product extract the quadrat roet: the ſame ſhall bee the Sine of the complement of the declination ſought for.

26. To find the *quantitie of the angles, which the circles of the* 12 *Howſes make with the Meridian.* Say

As the *Radius*
is to the Tang. of 60 degr. for the 11th, 9th, 5th, and third howers, or to the Tang. of 30 degr. for the 12th, 8th, 6th, and ſecond howres;
ſo is the Sine of the *complement of the Pole,*
to the tang. of the *compl. of any howſe with the Meridian.*

And note that on the Eaſterne part of the vpper hemiſphære, there are *three circles of Howſes,* the *Horoſcope,* which is alſo the Horizon, and next to that is the *circle of the* 12th *Howſe*, then the *circle of the* 11th *Howſe.* On the Weſterne part alſo, are *three circles of Howſes,* the *circle of the* 7th *Howſe,* which alſo is the Horizon, and next thereto the *circle of the* 8th *Howſe,* then the *circle of the* 9th *Howſe.* But the *circle of the* 10th *Howſe, is the very vpper Meridian it* ſelfe. *Contrary Howſes* are 1 and 7, 2 and 8 : 3 and 9 : 4 and 10 : 5 and 11 : 6 and 12.

27. Reſolue the whole time from the Noone laſt paſt into degrees (by multiplying the howers with their decimall parts by 15, according to Sect. 1) which adde vnto the right Aſcenſion of the Sunne: and you ſhall haue the right aſcenſion of the point of the Æquator in the vpper Meridian, which is called the *Right aſcenſion of Medium cœli.*

28. Adde 90 degrees to the Right aſcenſion of *Med. Cœli:* and it ſhall be the *degree of the Æquator then riſing vpon the Eaſt Horizon.*

29. If

29. If the *firſt quadrant of the Æquator doe ariſe*, the beginning of ♈ is diſtant from the meridian Eaſtward, ſo much as is the diſtance of the *Right aſcenſion of Med. cœli*, from 360. But if the *ſecond quadrant of the Æquator doe ariſe*, the beginning of ♈, is diſtant from the Meridian Weſtward, ſo much as is the diſtance of ☉, from the *Right aſcenſion of Med. cœli.*

And in both of them the lower angles of the Ecliptick with the Meridian, on the Eaſt ſide is obtuſe, and on the Weſt ſide acute: and the 90th degree of the Ecliptick, commonly called *nonageſimus gradus*, is on the Eaſt part.

30. If the *third quadrant of the Æquator doe ariſe*, the beginning of ♎ is diſtant from the Meridian Eaſtward, ſo much as is the diſtance of the *Right aſcenſion of Med. cœli* from 180. But if the *fourth quadrant of the Æquator doe ariſe*, the beginning of ♎, is diſtant from the Meridian Weſtward, ſo much as is the diſtance of 180, from the *Right aſcenſion of Med. cœli.*

And in both of them the lower angle of the Ecliptick with the Meridian, on the Eaſt ſide is acute, and on the Weſt ſide obtuſe: and on the 90th degree is on the Weſt part.

31. The *point of the Ecliptick culminant in the Meridian*, which is called *Medium cœli*, or *Cor cœli*, and is the *cuſpis of the* 10th *houſe*, may be found by Sect. 5.

32. The *declination* of the ſaid *culminant point*, may be found by Sect. 3. Wherefore alſo by adding or ſubducting that declination, to, or from the elevation of the Æquator,

Æquator, (which is the complement of the Pole) the *Altitude of Med. cœli* may be had.

33. The *Angle of the Ecliptick with the Meridian*, may be found by Sect. 7.

34. To finde the *Altitude of the 90 degr.* Or the *Angle of the Ecliptick with the horizon.*

As the *Radius*
is to the Sine of the compl. of the *altitude of Med: cœli.*
So is the Sine of the *angle of the Ecliptick with the Meridian,*
to the Sine of the compl. of *the angle sought for.*

35. To finde the *Azumith of 90 degr.* which is also the *Amplitude ortiue of the Ascendent, or Horoscopus.*

As the *Radius,*
is to the Sine of the *Altitude of Med. cœli.*
So is the tang. of the *Angle of the Ecliptick with the Meridian,*
to the tang. of the compl. of the *distance of that Azumith from the Meridian.*

36. To finde the *Horoscopus*, or *Ascendent degree of the Ecliptick*, Or the *Cuspis of the first house.*

The Distance of the Azumith of 90 degrees from the Meridian, is equall to the Amplitude ortiue of the Ascendent degree. Wherefore the Ascendent degree of

of the Ecliptic, may thence bee found, by Sect: 8, or 9. Or else thus

As the *Radius*
is to the Sine of the complement *of the angle of the Ecliptic with the Meridian:*
So is the tang. of the complement of the *altitude of Med. Cœli,*
to the tangent of the *distance of Med. cœli from the Ascendent degree.*

37. To find the *parts of the angle of the Ecliptic with the Meridian, cut with an arch perpendicular to the Circle of any of the Howses.* Say

As the *Radius*
is to the Sine of the compl. of the *altitude of Med. cœli:*
so is the tangent of the *circle of any Howse with the Meridian,*
to the tang: of the compl: of the *part of that angle, which is next the Meridian.*

Then subduct that part found out of the whole Angle for the remaining or latter part.

38. To find the *Distance of the cuspis of any howse, from Med: cœli.* Say

As the Sine of the compl. of the *later part of the angle of the Ecliptic with the Meridian,*
is to the Sine of the compl: of the *former part of that angle:*
So is the tang. of the *altitude of Med: cœli,*
to the tang: of the *distance of the cuspis of that Howse sought for.*

39. To find the *Altitude of the Pole above any of the circles of the Howses.*

First find out the Angle which the circle of the Howse proposed maketh with the Meridian, by Sect: 23: And then say.

As the *Radius*
is to the Sine of the *angle of the circle of the Howse with the Meridian*:
So is the Sine of the *height of the Pole above the Horizon of the place*,
to the Sine of the *height of the Pole above that circle of position.*

40. The *longitude*, and *latitude of any fixed Starre* being given, to find out the *Right ascension*, and *Declination* thereof.

The angle which the Circle of the Sunnes longitude maketh with the Meridian, at the Pole of the Ecliptic, I call the *Angle of longitude.*

And the angle which the Circle of the Sunnes Right ascension, maketh with the Meridian at the Pole of the world, I call the *Angle of right Ascension.* The condition and quantitie of which two angles, is thus found out.

In

In *Starres of the Northerne latitude*

If the longitude be in the I quadrant of the Ecliptic: ſubduct it out of 90: the remaines will bee the *angle of longitude*, acute. And the Angle of Right aſcenſion, being found, muſt be added vnto 270.

If the longitude bee in the II quadrant: ſubduct 90 out of it: the remaines will bee the angle of *longitude*, acute. And the Angle of right aſcenſion being found muſt be taken out of 270.

If the longitude bee in the III quadrant: ſubduct 90 out of it, the remaines will be the *angle of longitude*, obtuſe. And the Angle of right aſcenſion being found, muſt be taken out of 270.

If the longitude bee in the IIII quadrant, ſubduct it out of 90+360: the remaines will bee the *angle of longitude*, obtuſe. And the Angle of right aſcenſion being found, muſt be added vnto 270.

In *Starres of the Southerne latitude*

If the longitude be in the I quadrant, ſubduct 270 out of it + 360: the remaines will be the *angle of longitude*, obtuſe. And the Angle of right aſcenſion being found, muſt be taken out of 90.

If the longitude bee in the II *Quadrant*, ſubduct it out of 270: the remaines will bee the *angle of longitude*, obtuſe. And the Angle of right aſcenſion being found, muſt bee added to 90.

If the longitude be in the *III* quadrant: ſubduct it out of 270; the remaines will be the *angle of longitude*, acute. And the Angle of right aſcenſion being found, muſt bee added to 90.

If the longitude be in the *IIII* quadrant: Subduct 270 out of it: the remaines wil be the *angle of longitude*, acute. And the Angle of right aſcenſion being found, muſt bee ſubducted out of 90 * 360.

Then ſay,

As the *Radius*, or totall Sine,
is to the Sine of the complement, or exceſſe of
the *angle of longitude*:
So is the tang. of the compl. of the *latitude*,
to the tang. of the *firſt baſe*.

If the angle of longitude bee obtuſe; vnto the firſt baſe found, adde the greateſt declination deg. $23\frac{1}{2}$: and the ſumme ſhall be the *ſecond baſe*; and the angle of right aſcenſion ſhall be acute.

But if the angle of longitude be acute; out of the *firſt baſe* ſubduct the greateſt declination: and the remaines ſhall be the *ſecond baſe*. And the angle of right aſcenſion ſhall be obtuſe.

Or elſe out of the greateſt declination of the Sun, ſubduct the *firſt baſe*; and the remaines ſhall be the *ſecond baſe*: and the angle of right aſcenſion ſhall be acute.

Say againe,

As the Sine of the *ſecond baſe*,
is to the Sine of the *firſt baſe*:
So is the tang. of the *angle of longitude*,
to the tang. of the *angle of right aſcenſion*.

VVhence

VVhence by adding or ſubducting as was before deliuered in the conditions of thoſe angles, ſhall be giuen the Right aſcenſion of that Starre ſought for.

Laſtly ſay

As the tang. of the *ſecond baſe*,
is to the *Radius*:
So is the Sine of the compl. or exceſſe of the
angle *of right aſcenſion*
to the tang. of *Declination*.

VVhere note that if the *ſecond baſe* exceede 90 degr: the declination found, ſhall not be of the ſame kinde, that the latitude is, but in the contrary Hemiſphære.

A Table of the Right Aſcenſions, and Declinations of 40 of the cheifeſt fixed Starres, Calculated for the yeare of our Lord. 1650.

Names of Starres	Right Aſcenſion		Declination			mag.
The Polar Starre	7°	47′	80°	27′	N	2
Andromedaes girdle	12	32	33	48	N	2
the former horne of ♈	23	38	17	37	N	4
the bright ſtarre in the head of ♈	26	56	21	48	N	3
the iawe of the Whale	41	03	2	42	N	2
Meduſaes head	41	27	29	35	N	3
the eye of the Bull	64	00	15	46	N	1
the Goat ſtarre	72	44	45	35	N	1
the former ſhoulder of Orion	76	38	4	59	N	2
the latter ſhoulder of Orion	84	07	7	18	N	2
the great dogge ſtarre	97	27	16	13	S	1
the higher head of ♊	108	01	32	35	N	2
the leſſer Dogge ſtarre	110	17	6	06	N	2
the lower head of ♊	111	00	28	49	N	2
the Cribb, or Manger	125	4	20	52	N	*Neb.*
the heart of Hydra	137	39	7	10	S	2
the heart of the Lion	147	27	13	39	N	1
In the loynes of the Lion	163	54	22	26	N	2
In the tayle of the Lion	172	49	16	32	N	1
In the girdle of Virgo	189	32	5	20	N	3

Names of Starres	R. Ascen.		Declination			mag.
Aliot	189°	36	57	53	N	2
Vindemiatrix	191	15	15	51	N	3
Spica Virginis	196	44	9	17	S	1
Arcturus	209	56	21	34	N	1
the Southerne ballance	217	5	14	32	S	2
the Northerne ballance	224	31	8	2	S	2
the bright ſtar in the ſerpents neck	231	49	7	35	N	3
the heart of Scorpius	242	4	25	34	S	1
the head of Hercules	254	40	14	51	N	3
the head of Ophiucus	259	41	12	52	N	3
bright ſtarre in the Harp	276	17	38	30	N	1
bright ſtarre in the Vultur	293	27	8	1	N	2
vpper horne of ♑	299	30	3	32	S	3
the left hand of ♒	307	10	10	43	S	4
the left ſhoulder of ♒	318	18	7	2	S	3
the mouth of Pegaſus	321	49	8	18	N	3
the right ſhoulder of ♒	326	59	1	58	S	3
Fomahant	339	29	31	23	S	1
In the vpper wing of Pegaſus	241	53	13	21	N	2
In the tip of the wing of Pegaſus	358	52	13	15	N	2

41. The *longitude*, and *latitude of any two Starres* being giuen, to finde their *diſtances*.

If the Starres haue both the ſame longitude, differing onely in latitude; the difference of latitude, is the diſtance of the ſtarres.

And if they differ onely in longitude hauing the ſame latitude; Say

As the *Radius*,
is to the Sine of *halfe the difference of longitude*:
So is the Sine of the compl. of the latitude giuen,
to the Sine of *halfe the diſtance of the Starres*.

But if they differ both in longitude, and latitude, whether the latitudes be both of the ſame kinde, or one Northerne, and the other Southerne. Take the difference of both the ſtarres, from the pole of the Ecliptick, toward which the ſtarre hauing the greater latitude is. And ſay,

As the tang. of the compl. of the *leſſer diſtance from the pole*,
is to the *Radius*:
So is the Sine of the compl. or *exceſſe of the difference of longitudes*,
to the tang. *of the firſt baſe*.

Take this *firſt baſe* out of the greater diſtance from the pole, and the remaines ſhal be the *ſecond baſe*, Then ſay,

As the Sine of the compl. of the *firſt baſe*,
is to the Sine of the compl. of the *ſecond baſe*:
So is the Sine of the compl. of the *leſſer diſtance from the pole*,
to the Sine of the compl. of the *diſtance of the two Starres*.

If

If any man will take paines to calculate (by this last Rule) the distances of some noted starrs of the first, second, and third magnitudes, round about the heauens, which are not aboue 5, or 6 degrees, at the most, one from the other: and shall keepe them written in his booke: they may serue as a Rule, or Instrument, whereby he may reasonably estimate with his eye, the distance of any Planet, or Comet, or other apparition from a knowen fixed starre, not very farre remote: by comparing the distance which hee would know with some of those knowen distances which he shall find, either to be equall, or else to haue some proportion thereto.

42. The *longitude*, and *latitude of any two Cities* being giuen, to find their *distance*.

The manner of the operation is the very same with the former, vnto which therefore I referre the Reader: onely I will note, that in the heauens, the longitude and latitude is taken in respect of the Ecliptic, which being the way of the Sunne, all the starrs in their proper motion, haue reference vnto it, as vnto their measure and rule. But in the Earth the principall Circle is the Equinoctiall, diuiding it into the Northerne, and Southerne hemisphæres. And therefore in the earth, the longitude, and latitude is reckoned by the Æquinoctiall.

The distance of two places vpon the Earth, being found in degrees, may bee conuerted into English miles, by taking 60 miles for euery degree, and one mile for euery minute.

43. To find *at what hower a fixed starre commeth into the Meridian any day.*

Seeke the Right ascension of the Sunne, for that day, by Sect: 5; and subduct it out of the Right ascension

of the Starre. And reduce the degrees remaining into howers, by Sect: 1. The ſame ſhall ſhew how long time from the Noone before, the ſame ſtarre ſhall come into the Meridian.

Wherefore if at any time of the night, a Starr whoſe Right aſcenſion is knowne, be in the Meridian, the hower of the night is eaſily found.

44. The *height of any knowne Starre* aboue the Horizon, being by any means given, to find the *hower of the night.*

Firſt ſeeke out the hower of that ſtarrs comming into the Meridian the ſame day, by Sect. 43. Againe ſeeke out the horary diſtance of that ſtarr from the Meridian, according to Sect: 19. And then if the ſtarr bee on the Eaſt ſide, not yet come to the Meridian, take the difference of thoſe two numbers; but if the ſtarre bee paſt the Meridian, take the Summe of them, for the houre of the night.

45. The *height of the Pole* being given to find the *comming of any fixed Starre, in the due Eaſt, or Weſt.* Say

As the *Radius*
is to the tang: of the *ſtarres declination:*
So is the tang: of the *compl: of the Pole,*
to the Sine of the compl. of the *Starres horary diſtance from the Meridian.*

46. The *height of the Pole* being giuen, to find the *Altitude of any fixed ſtarr aboue the Horizon, being due Eaſt or Weſt.* Say,

As

As the Sine of the *height of the Pole,*
is to the *Radius*:
so is the Sine of the *Starrs declination,*
to the Sine of the *Altitude, at due East or West.*

47. By the *Altitudes of any two knowne fixed starrs taken when they are both in the same Azumuth, to find the height of the Pole.*

First say,

As the Sine of the *difference of the starrs altitudes,*
is to the Sine of the *difference of their Right ascensions*:
so is the sine of the *neerer starrs distance from the apparent Pole,*
to the Sine of *an angle to be kept.*

Againe compare the furtheſt ſtarrs diſtance from the Pole with the diſtance from the *Zenith,* and ſay

As the *Radius*
is to the Sine of the compl. of the *Angle kept*:
ſo is the tang: of the *leſſer of the compared arches,*
to the tang: of the *firſt baſe.*

Subduct the *firſt baſe* out of the greater of the two compared arches; and the remaines ſhall bee the *ſecond baſe.*

Then laſtly ſay,

As the Sine of the complement *of the firſt baſe,*
is to the Sine of the compl. *of the ſecond baſe*:
ſo is the Sine of the compl. of the *leſſer of the two compared arches,*
to the Sine of the *height of the Pole.*

48. To find out the *horizontall Parallax of the Moone.*

First the distance of the Moone from the Center of the earth must be knowne in Semidiameters of the earth: which vnto them that are acquainted with the Theorie of the Planets, is not very difficult. And whereof peraduenture, I may hereafter teach the practise, by most easie and exact instruments, which I haue long since framed.

Say,

As the *distance of the Moone, from the center of the earth,*
is to the *Semidiameter of the earth:*
So is the *Radius,*
to the Sine of the *Moones horizontall parallax in that distance.*

49. The *horizontall Parallax of the Moone* being given, to find *her Parallax in any apparent altitude.*

As the *Radius,*
is to the Sine of the *altitude of the Moone:*
so is the Sine of the *horizontall Parallax,*
to the Sine of the *Parallax in that altitude.*

50. The *place of the Moone in the Ecliptic* hauing little or no latitude (as in the Eclips of the Sunne) together with *her Parallax of altitude* being giuen, to find the *Parallaxes of her longitude,* and *latitude.*

If the Moone bee in the 90th degree of the Ecliptic: shee hath no Parallax of longitude, and the Parallax of latitude, is the very Parallax in that altitude.

But

But if the Moone be not in the 90th deg. say,

As the *Radius*
is to the tang. of the *angle of the Ecliptick with the horizon:*
So is the Sine of the compl. of the *distance of the Moone, from the Ascendent, or descendent degree of the Ecliptick,*
to the tang of the compl. of the *angle of the Ecliptick with the Azumith of the Moone.*

Againe say,

As the *Radius*
is to the Sine of that *angle:*
So is the *Parallax of the Moones altitude,*
to the *Parallax of her latitude.*

Lastly say,

As the *Radius*
is to the Sine of the compl. of that *former angle:*
So is the *Parallax of the Moones altitude,*
to the *Parallax of her longitude.*

which is to bee added to the true motion of the Moone, if she be on the East part of the 90th degree of the Ecliptick: or to be subducted out of it, if she be on the West part.

Many other *Astronomicall* and *Geographicall problemes* might be added. But because it is impossible to set downe all, which may be of vse, at some time or other: I haue in the next Chapter deliuered briefly the *doctrine of triangles* fitted vnto practise: with all the seuerall cases belonging thereto.

CHAP. XIII.

Of Trigonometria, *or the manner of calculating both* Plaine, *and* Sphæricall triangles. *And first concerning certaine generall notions, and rules necessary thereto.*

IN euery triangle both Plaine, and Sphærical, the greater side subtendeth the greater angle. And the greater angle hath the greater side opposite vnto it. Also the greater angle lyeth to the lesser side, and the greater side hath the lesser angle lying vnto it.

In euery plaine triangle, any two angles being giuen, the third is also giuen: and one of the angles being giuen, the summe of both the other two is giuen. For all the three angles together, are equall to two right angles, that is to 180 degrees.

In a plaine rectangle triangle, one of the acute angles is the complement of the other. Where note that when the complement is named without any other addition, it is meant of the arch, which is wanting of a quadrant of that circle, or 90 degrees. In like manner the excesse is meant of the arch, which is aboue a quadrant. But when it is said the complement to a semicircle, it is vnderstood of so many degrees as will make vp 180.

But in a Sphæricall rectangle triangle, one of the oblique angles is alwayes greater then the compl: of the other.

If two arches together make vp a Semicircle, the excesse of the greater arch, is equall to the complement of the lesser.

The

The ſame Right Sine, and the ſame Tangent, and Secant, doth belong both to the arch it ſelfe, and alſo to the complement of it to a Semicircle. But their verſed Sines differ: For the verſed Sine of an arch leſſe then a quadrant, is equall to the difference of the *Radius*, and the Sine of the complement of that arch: and the verſed Sine of an arch greater then a quadrant, is equall to the ſumme of them. And the verſed Sine is thus found out, As the *Radius* is to the Sine of halfe the arch; ſo is the Sine of that halfe arch, to halfe the verſed Sine of the whole arch.

In a right angled triangle both Plaine, and Sphæricall, one of the ſides containing the right angles, is called the Baſe, and the other the Cathetus: and the ſide ſubtending the right angle is the Hypotenuſa. And know that euery rectangled triangle, is moſt fitly noted with the letters *ABC*; ſo that *BA* may be the Baſe, and *CA* the Cathetus, and *BC* the Hypotenuſa: and *B* the angle at the baſe, and *C* the angle at the Cathetus, and *A* the right angle. Likewiſe euery oblique-angled triangle with the letters *BCD*; ſo that out of the angle *C* a perpendicular *CA*, being let downe, it may in the baſe *BD* diſtinguiſh the two caſes *BA* and *DA*, which are the baſes of the two particular triangles into which it is cut. And in noting the triangles with letters, obſerue diligently, that if any angle be giuen together with one of the ſides including it, the ſame angle be noted with *B*; and the ſide with *BC*.

If both the angles at the Baſe *BD* be acute, the perpendicular *CA* ſhal fall within the triangle: And *BD*=*BA*+*DA* that is *BD* is equall to the ſumme of *BA* and *DA*. And if the angle *B* be obtuſe, the perpendicular *CA* ſhall fall without the triangle, beyond the obtuſe angle *B*: And *BD*=*DA*—*BA*, that is *BD* is equall to the exceſſe of *DA* aboue *BA*: or if the angle *B*. be obtuſe, the perpendicular *CA*: ſhall fall without the triangle beyond the obtuſe

obtuſe angle $\mathcal{D}$: And $BD = BA - \mathcal{D}A$, that is BD is equall to the exceſſe of BA aboue $\mathcal{D}A$. The leſſer caſe being ſtill taken from the greater angle.

And note that this ſigne $*$, or *pl* (that is *plus*) ſheweth that the magnitude before which it is ſet, is affirmed and poſitiue in nature; and therefore to bee added. And that this ſigne — or *mi* (that is *minus*) ſheweth the magnitude before which it is placed, is denied, and priuatiue in nature; and therefore to be ſubſtracted, as you may ſee in thoſe former examples.

Againe, ſome magnitudes are taken ſeuerally and apart: as $s\,BA$, that is the Sine of the Baſe; $s\,co\,BC$, that is the Sine of the complement of the Hypotenuſa; $t\,B$, that is the tangent of the angle ABC at the baſe; $t\,co\,C$, or $t\,co\,ABC$, the tangent of the complement of the angle at the Cathetus: So alſo $\sqrt{q}\,Z$, that is the quadrat ſide of the plaine Z. And ſome magnitudes are taken vniuerſally, and then they are included in pricks: as $s.\frac{DC+BD-BC}{2}.$ that is the Sine of halfe the arch, which is compoſed of the ſumme of the two arches DC, and BD, abating thereout the arch BC. So alſo $\sqrt{q}: Z * X:$ that is the quadrat ſide of the two plaines Z and X put together: alſo $\sqrt{q}: \mathcal{Q}$ in $R:$ Or $\sqrt{q}: \mathcal{Q} * R:$ that is the quadrat ſide of a rectangular plaine, the two ſides whereof are the lines $\mathcal{Q}$ and R, or ſome fourth proportionall already found, and the *Radius*, or Semidiameter, which is the totall Sine. For by the ſigne in, or $*$, I vſe to expreſſe multiplication.

When any triangle is giuen to be reſolued by Trigonometrie, note the parts thereof (either ſides or angles) which are giuen and knowne, with a little line drawne croſſe each ſuch part: and note the vnknowne part which is ſought for with a little circle.

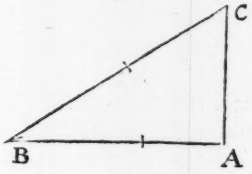
And

And if a triangle Sphæricall (bee it right angled or oblique-angled) propoſed hath two ſides each of them ſeuerally greater then a quadrant: you ſhall in reſoluing thereof, keepe the leaſt ſide with the leaſt angle oppoſed to it: and for the two other both ſides, and angles, take the complements of them to a ſemicircle.

Laſtly, if a triangle with all the three angles giuen, be required to be conuerted into a triangle hauing the three ſides giuen. You ſhall for the greateſt angle of the triangle propoſed, & for the greateſt ſide ſubtending it, take the complements to a ſemicircle; keeping the other two leſſer angles, with their ſubtendent ſides as they are.

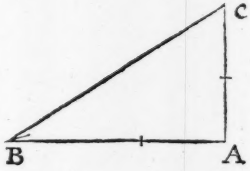
THE CALCVLATION OF PLAINE *right angled-triangles.*

I.



BC . BA :: R . *s co* B (*s* C) :

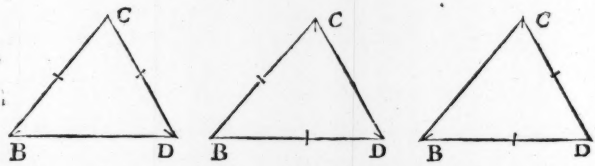
II.



BA . CA :: R . *t* B (*t co* C) :

Of plaine Oblique-angled triangles.

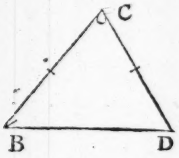
III.



s B . DC :: *s* D . BC :: *s* C . BD .

and here it is necessary to be knowne, whether the angle sought for be greater, or lets then a right angle, or 90 deg.

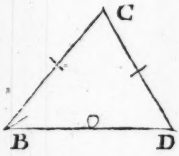
First



IIII.

First seek the angle D, by the III; then both the angles B and D being subducted out of 180, you shall haue

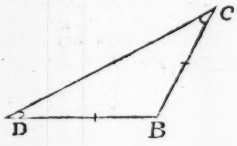
$$180 - B - D = C .$$



V.

First seeke the angle D, by the III; then both the angles B and D being subducted out of 180, Say

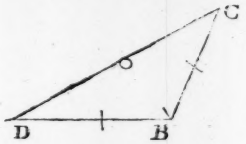
$$s B . DC :: s : 180 - B - D . BD .$$



VI.

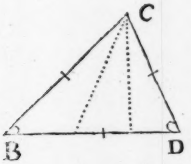
Let the side BD be greater then the side BC:
First, $BD + BC . BD - BC :: s \frac{180-B}{2} . sQ$:
then for the other two angles:
$\frac{180-B}{2} + Q =$ the greater . $\frac{180-B}{2} - Q =$ the lesser.

VII.



Let the side BD be greater then the side BC:
First, the angles C and D are to be sought, by the VI. and the side DC, by the III.

VIII.



Take the greatest side BD for the base: and let the side BC, be greater then the side DC. First say,

BD . BC+DC :: BC—DC . Q (*viz* BD—2DA).

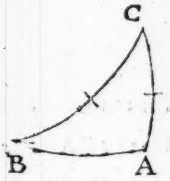
then $\frac{BD+Q}{2}$ BA . $\frac{BD-Q}{2}$ DA.

Next seeke the angles B and D, by the III:
Lastly 180—B—D=C.

THE

THE CALCVLATION OF Sphæricall right-angled, and quadrantall triangles.

I.

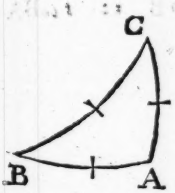


R . *s*B :: *s*BC . *s*CA .

II.

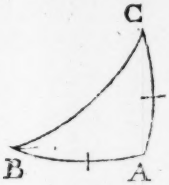
R . *s*B :: *s co*BA . *s co*CA .

III.



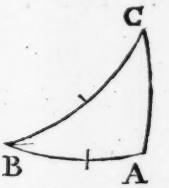
R . *s co*BA :: *s co*CA . *s co*BC .

IIII.



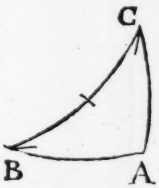
R . *s*BA :: *s*B . *s*CA .
R . *s co*B :: *s*CA . *s*BA .
R . *s*BA :: *s co*CA . *s co*B .

V.



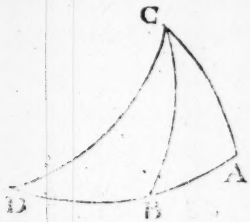
R . *s co*B :: *s*BC . *s*BA .
R . *s co*BC :: *s*BA . *s co*B .
R . *s co*B :: *s co*BA . *s co*BC .

VI.



R . *s co*BC :: *s*B . *s co*C .

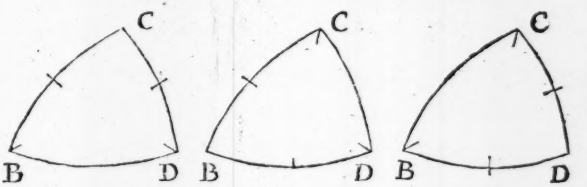
If



VII.

If a triangle BCD be quadrantall, hauing one ſide BC equall to a quadrant; vpon the pole D deſcribe an arch of a great circle CA, cutting the ſide DB extended in A: and ſo making a right-angled triangle ABC without the other. This outward right-angled triangle ſhall be reſolued in ſteed of the quadrantall propoſed.

Of Sphæricall Oblique-angled triangles.



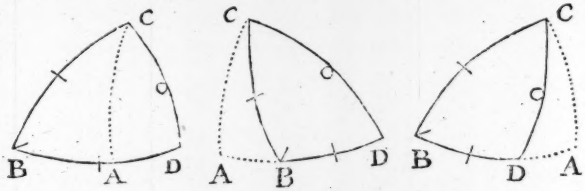
VIII.

$\sin B \cdot \sin DC :: \sin D \cdot \sin BC :: \sin C \cdot \sin BD$.

and in theſe it is neceſſary to bee knowne whether the terme ſought for be greater then a quadrant, or not. The ſame alſo is to be knowne in the tenne rules next following, if the ſides BC and DC are both giuen.

First,

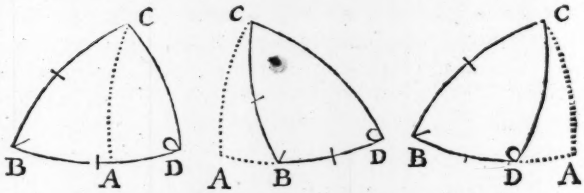
IX



First, R . *sco*B :: *s*BC . *s*BA :

then, *sco*BA . *sco*DA :: *sco*BC . *sco*DC :

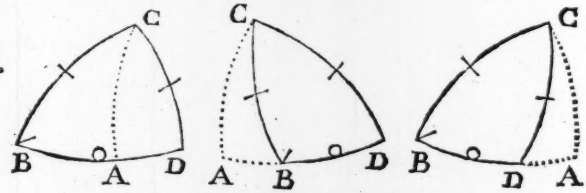
X.



First, R . *sco*B :: *s*BC . *s*BA :

then *s*DA . *s*BA :: *s*B *s*D .

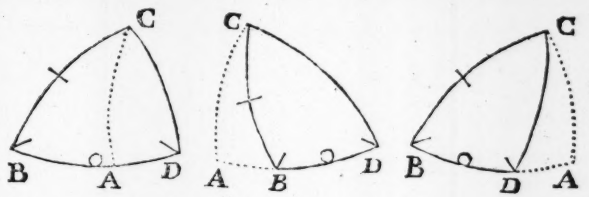
XI.



First, R : *sco*B :: *s*BC : *s*BA

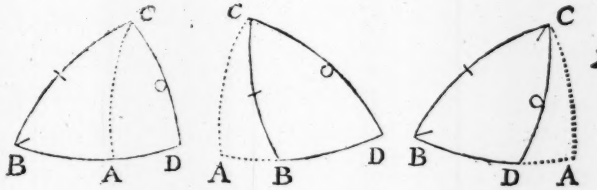
then, *sco*BC . *sco*DC :: *sco*BA . *sco*DA :

First,



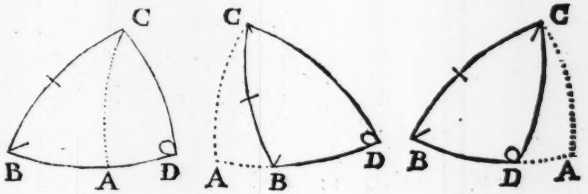
XII.

First, R . *s co*B :: *t*BC . *t*BA :
then, *t*D . *t*B :: *s*BA . *s*DA :



XIII.

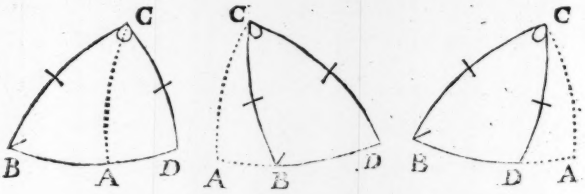
First, R . *s co*BC :: *t*B . *t co*BCA :
then, *s co*DCA . *s co*BCA :: *t*BC . *t*DC :



XIIII.

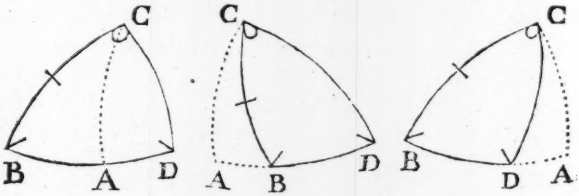
First, R . *s co*BC :: *t*B . *t co*BCA :
then, *s*BCA . *s*DCA :: *s co*B . *s co*D :

XV.



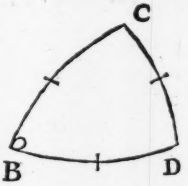
First, R . *s co*BC :: *t* B . *t co*BCA .
then, *t* DC . *t* BC :: *s co*BCA . *s co*DCA .

XVI.



First, R . *s co*BC :: *t* B . *t co* BCA .
then, *s co* B . *s co*D :: *s* BCA . *s* DCA .

XVII.

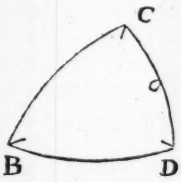


First, R . *s* BD :: *s* BC . QI .
then, QI . *s* : $\frac{DC+BD-BC}{2}$ :: *s* : $\frac{DC+BC-BD}{2}$. QII .

See what QII cutteth in the fift circle, which is of equall diuiſions: and thereto adde the *Radius*, by ſetting

1 before

1 before that number. Diuide the whole into two equall parts: and reckoning one halfe in that fift circle, set the Index to it, and it shall in the first circle cut the Sine of halfe the angle *B*.



XVIII.

If all the three angles be giuen: conuert the triangle into another hauing all the three sides giuen: and resolue the same for the triangle proposed.

CHAP. XIIII.

Of the Nocturnall Dials.

THere are in the Inſtrument, two ſeuerall *Nocturnall Dyals.* The innermoſt of them is fitted to the ſtarre in the rump of the great Beare, commonly called *Aliot.* The other is compoſed of 12 ſeuerall ſtarres: whoſe names you ſhall finde written within, neere to the center.

The outermoſt circle of the Nocturnall Dyall is diuided into twiſe 12 houres: each houre being ſubdiuided into quarters, and are noted with figures belonging to the houres, as may be ſeene in the Inſtrument.

The middlemoſt circle of the Nocturnall is diuided into 12 moneths, hauing their names written: each moneth being diſtinguiſhed into tenth dayes with longer lines; and into fift dayes with ſhorter lines. And if the Inſtrument be large enough, each day of the monethes throughout the yeare, may be noted with pricks.

In the innermoſt circle are the diuiſions and names of 12 fixed ſtarres: which are theſe.

The bright ſtar in the head of ♈.	Spica Virginis
the Bulls eye	the North ballance
the latter ſhoulder of Orion	the head of Ophiuchus
the little dogge	the heart of the Vultur
the heart of the Lyon	the mouth of Pegaſus
the tayle of the Lyon	the tip of the wing of Pegaſus.

To finde out the houre of the night by Aliot.

Seeke the day of the moneth in the annuall circle of the Nocturnall: and apply the Index thereto: marke what houre it cutteth, in the houre circle. Remember this houre

houre for all that day: then at night when you would finde out the houre, hold vp your Inſtrument by the handle, and moue it vp and downe, till you ſee the pole ſtarre through the middle hole, and the ſtarre called *Alioth* by the limbe: Set the Index or labell to *Alioth*, and marke what houre the labell cutteth, for if vnto this houre you adde the houre kept in minde for that day, the ſumme of both ſhall giue you the true houre of the night: ſo that you caſt out 12 houres, from the ſaid ſumme if it ſhall chance to be more.

Example. If on the 15th of Nouember you would find the houre of the night by the ſtarre *Alioth*. Apply the Index to the day of the moneth, and it will cut in the houre circle 8 and an halfe: then ſuppoſe the Index being ſet to the ſtarre *Alioth*, as aboue taught, doth cut in the houre circle 10. theſe two numbers being added together, the ſumme wil be $18\frac{1}{2}$, out of which ſubduct 12; and the remaines $6\frac{1}{2}$, will be the true houre of the night.

To finde out the houre of the night by the Inner Nocturnall Dyall.

To performe this it is neceſſary that you know the true Meridian of the place wherein you are, and can finde it out by night, which you may thus doe. Hauing a Meridian line drawne in ſome window, or other conuenient place (as is ſhewed in the Second part of this booke, *Vſe* 19) ſtick vp therein a long needle perpendicularly, and watch till the Sunne caſteth the ſhadow of the needle, vpon the Meridian line. Or elſe in a true Sunne Dyall obſerue when the ſhadow falls iuſt on 12 a clock, for then is the Sunne in the Meridian. Wherefore goe inſtantly into ſome place about your houſe where you may ſee ſome marke, either a chimney, or the corner of an houſe, or elſe ſome tree, or ſuch like, directly betweene you and the Sunne: then haue you the true Meridian.

Or otherwiſe you may in a cleare night goe into ſome plaine place neere your houſe, and ſetting vp a ſtraight pole perpendicularly on the ground, goe a good diſtance from it Southwards; and then moue vp and downe, till you ſee the top of your pole, directly betweene your eye, and the North polar ſtarre: then ſet vp another pole perpendicularly betweene your feet, ſo that both your poles, and the Polar ſtarre, may be in one right line. And then going backe againe to your firſt pole, looke what knowne ſtarre is directly ouer your laſt pole, for that ſtarre is in the Meridian. You may therefore inſtantly goe to ſome conuenient place, and take a marke whereby you may at all times know the Meridian as is afore taught.

When therefore at any time of the night you would know what a clocke it is, goe to that place where you ſtood, and looking directly ouer your marke, ſee if any of the 12 fixed ſtarres, bee in the Meridian; or if none of them be therein, obſerue which two of them are on either ſide thereof, and what part of that ſpace is in the Meridian. Then goe into the light, and take your inſtrument, and ſet the Index to that ſtarre, or point which you ſaw in the Meridian: marke what houre it cutteth, for that ſame houre being added to the houre, which the day of the moneth ſheweth, ſhall giue you the true houre of the night: ſo that you caſt out 12 houres, from the ſaid ſumme, if it ſhall chance to be more.

Example. Suppoſe the fifth of December, that the middle point of the ſpace betweene the bright ſtarre in the head of *Aries*, and the *Bulls eye*, bee in the Meridian. Set the Index to the middle point of the ſpace betweene thoſe two ſtarres in the Inſtrument: and it will cut in the houre circle 2 and an halfe: then againe ſet the Index to the fifth of December, and in the houre circle it wil cut 7: which

which added vnto 2 and an halfe, giueth 9 and an halfe, for the true houre of the night.

Another example. Suppoſe the 19th of December, that one third part of the ſpace betweene the *Bulls eye*, and the *right ſhoulder of Orion*, be in the Meridian. Set the Index to one third part of that ſpace in the Inſtrument, and it will cut in the houre circle 4 and halfe a quarter almoſt: againe, ſet the Index to the 19th of December, and in the houre circle it will cut 6, which being added vnto 4, and halfe a quarter almoſt, giueth 10 and almoſt halfe a quarter for the houre of the night.

THE

THE SECOND PART OF THIS BOOKE.

Shewing the vſe of the Second ſide *of the Inſtrument, for the working of moſt queſtions, which may be performed by the* Globe: *And the* declination of Dyals, *vpon any kinde of Plaine.*

VPon the *ſecond ſide* of the Inſtrument, is delineated the *proiection of the vpper Hemiſphære vpon the plaine of the Horizon*: The *Horizon* it ſelfe is vnderſtood to bee the innermoſt circle of the limbe: and is diuided on both ſides, from the points of *Eaſt*, and *Weſt* into degrees, noted with 10, 20, 30, &c. vnto 90. And the center of the Inſtrument is the *Zenith*, or *Vertical point*.

Within the *Horizon*, the middle ſtraight line, or Diameter pointing North and South, is the *Meridian*, or 12 a clock line: and the other ſhort arching lines, on both ſides of it are the *houre lines*, diſtinguiſhed accordingly by their

figures. Theſe houre lines ſhould indeede bee drawne through the whole plaine, croſſing one another in the Pole of the world: but that the Inſtrument may be more faire, they are onely drawne ſhort.

And becauſe diuers excellent vſes, doe require the totall delineation of the houre circles, I haue in a ſeuerall paper, inſerted intirely, both the houre lines, and alſo two other circles betweene them, containing euery one fiue degrees. (But if the Inſtrument were large enough to receiue them, it were beſt it euery degree had his circle: and ſo euery 15 circle ſhould bee an houre line.) And of the parallels, there needes no more, but the Æquinoctial, and both the Tropics.

For as much as there will be great vſe of this paper Inſtrument; I haue in the 24 Vſe ſhewed the manner of making it: ſo that any that is ingenuous, and ready handed may himſelfe delineate one ſufficient enough to ſerue his turne, for any eleuation.

The two arches which croſſe the houre lines meeting on both ſides in the points of interſection of the ſixe a clocke lines with the Horizon, are the *two Semicircles of the Ecliptick*, or Annuall circle of the Sunne: the vpper of which arches ſerueth for the *Summer halfe yeare*, and the lower for the *Winter halfe yeare*: and are therefore diuided in 365 dayes: which are alſo diſtinguiſhed into 12 moneths with longer lines, hauing their names ſet downe: and into tenthes, and fifthes with ſhorter lines: and the reſt of the dayes with pricks: as may plainly bee ſeene in the Inſtrument.

And this is for the ready finding out of the *place of the Sunne* euery day: and alſo for ſhewing of the *Sunnes yearely motion*: becauſe by this motion the Sunne goeth round about the heauens in the compaſſe of a yeare, making

ing the *foure parts*, or *ſeaſons* thereof. Namely the *Spring*, in that quarter of the Ecliptick which beginneth at the interſection on the Weſt ſide of the Inſtrument, and is therefore called the *Vernall interſection*. Then the *Summer* in that quarter of the Ecliptick which beginneth with the interſection of the Meridian in the higheſt point next the Zenith. And after that *Autumne* in that quarter of the Ecliptick, which beginneth at the interſection on the Eaſt ſide of the Inſtrument, and is therefore called the *Autumnall interſection*. And laſtly, the *Winter* in that quarter of the Ecliptick, which beginneth at the interſection with the Meridian, in the loweſt point next the Horizon.

But beſides this yearely motion, the Sunne hath a *Diurnall* or *dayly motion*, whereby it maketh day and night with all the diuerſities, and inequalities thereof: which is expreſſed by thoſe other circles drawne croſſe the houre lines: the middlemoſt whereof being groſſer then the reſt, meeting with the Ecliptick in the points of the *Vernall*, and *Autumnall interſections*, is the *Æquinoctiall*: and the reſt on both ſides of it, are called the *Parallels*, or *Diurnall arches of the Sunne*: the two outermoſt whereof are the *Tropics*, becauſe in them the Sunne hath his furtheſt *digreſſion*, or *Declination* from the Æquinoctiall, which is degrees 23½: and thence beginneth againe to returne to the Æquinoctiall. The vpper of the two Tropics next the center (in this our Northerne Hemiſphære) is the *Tropick of Cancer*: and the Sunne being in it, is higheſt into the North, making the longeſt day of Summer. And the lower nex the Horizon, is called the *Tropick of Capricorne*; and the Sunne being in it, is loweſt into the South, making the ſhorteſt day of Winter.

Betweene the two Tropics, and the Æquinoctiall infinite ſuch *parallel circles* are vnderſtoode to bee

contained: for the Sunne is what point ſoeuer of the Ecliptick it is caried, deſcribeth by his lation, a circle parallel to the Æquinoctial. Yet thoſe parallels which are in the Inſtrument, though drawne but to euery ſecond degree of Declination, may be ſufficient to direct the eye, in imagining and tracing out, through euery day of the whole yeare in the Ecliptick, a proper circle, which may be the *Diurnall arch of the Sunne* for that day. For vpon the right eſtimation of that imaginary parallel, doth the manifold vſe of this Inſtrument eſpecially rely: becauſe the true place of the Sunne, all that day, is in ſome part, or point of that circle. Wherefore for the better conceiuing, and bearing in minde thereof, euery fift parallel, is herein made a little groſſer then the reſt.

I Vſe. *And thus by the eye, and view only, to behold and comprehend* the courſe of the Sunne, *both for his* Annuall *and* Diurnall motion, *may be the firſt vſe of this Inſtrument.*

II Vſe. *To take the* height of the Sun *aboue the Horizon.*

Set vp the pinne, (which is therefore made fit for the hole at the center) perpendicular in the center: and put the Indices on both ſides, downe vpon the Meridian, that they with their waight, may not ſway the Inſtrument any way as it hangeth: then with a threed put into the hole aboue in the handle, hang it perpendicularly, bearing the edge toward the Sunne, that the pinne may caſt a ſhadow, vpon the degrees in the limbe: for that degree which the ſhadow of the pinne cutteth in the limbe, is the height of the Sunne aboue the Horizon, at that preſent.

III Vſe.

III. Vſe. *To find the* Declination of the Sunne *euery day.*

Looke the day of the moneth propoſed in the Ecliptic, and marke how many degrees the prick ſhewing that day, is diſtant from the Equinoctiall, either on the Summer, or Winter ſide, *viz.* North, or South.

Example. I. What will the Declination of the Sunne be, vpon the 11th day of *Auguſt*? Looke the 11th day of *Auguſt*: and you ſhall find it in the ſixt Circle aboue the Æquinoctiall: now becauſe each Parallel, ſtandeth (as hath beene ſaid before) for 2 degrees, the Sunne ſhall that day decline North-wards 12 degrees.

Example. II. What Declination hath the Sunne, vpon the 24th day of *March*? Looke the 24th day of *March*, and you ſhall find it, betweene the ſecond, and third Northerne parallels, as it were an halfe and one fift part more of that diſtance from the ſecond: reckon therefore 4 degrees for the two Circles, and one degree for the halfe ſpace: ſo ſhall the Sunnes declination bee 5 degrees, and about one fift part of a degree Northward, that ſame day.

Example. III. What Declination hath the Sunne vpon the 13th day of *Nouember*? Looke the 13th day of *Nouember*, and you ſhall find it below the Equinoctiall, tenne parallels and about one quarter, which is 20 degrees, and an halfe South-wards. So much is the Declination. And according to theſe examples iudge of all the reſt.

IIII. Vſe. *To find the* Right aſcenſion of the Sunne *euery day.*

Imagine an hower line through the day of the moneth giuen, and marke in what point it will croſſe the

Æquinoctiall: then lay a Ruler, or a ſtreight Scroule of paper, to the Pole of the world (noted in the Inſtrument with *P W*) and that ſame point. For the Ruler ſhall in the innermoſt Circle of the limbe, of the South ſide, cut the Right aſcenſion of the Sunne for that day, to be reckoned from the Weſt, to the point of interſection, for the firſt, or vpper Semicircle of the Ecliptic: or from the Eaſt together with 180, for the ſecond, or lower Semicircle of the Ecliptic.

V. Vſe. *To find the* longitude of the Sunne, *or in what* degree of the Signe *he is euery day.*

The Pole of the firſt Semicircle of the Ecliptic is noted *P I.* and the Pole of the ſecond Semicircle is noted *P I I.* Lay a Ruler, or a ſtreight Scroule of paper, to the day of the moneth, and the proper Pole of the Semicircle of the Ecliptic, in which it is: for the Ruler ſhall in the innermoſt Circle of the limbe, on the South ſide, cut the degree of the Sunnes place in the Ecliptic, reckoning it in the ſame manner as you did in finding the Sunnes Right aſcenſion: and the Arch thus found is called the longitude of the Sunne. which may bee expanſed into ſignes, by reckoning on the limbe, from the Weſt to South ♈, ♉, ♊, and from South to Eaſt ♋, ♌, ♍: then backe againe from Eaſt to South ♎, ♏, ♐; and laſtly from South to Weſt ♑ ♒, ♓, allowing 30 degrees, for each of thoſe twelue ſignes.

VI. Vſe. *To find the* Diurnall Arch, *or* Circle of the Sunnes courſe *euery day.*

The Sunne euery day by his motion (as hath beene ſaid) deſcribeth a Circle parallell to the Æquinoctiall, which is either one of the Circles in the Inſtrument, or ſome

ſome-where betweene two of them. Firſt then ſeeke the day of the moneth; and if it fall vpon one of thoſe Paralels, that is the Circle of the Sunnes courſe that ſame day: But if it fall betweene any two of thoſe Parallels, imagine in your minde, and eſtimate with your eye, another Parallel through that point, betweene thoſe two Parallels, keeping ſtill the ſame diſtance from each of them.

As in the firſt of the three former Examples, The circle of the Sunnes courſe, vpon the 11th day of *Auguſt*, ſhall be the very ſixt Parallel aboue the Æquinoctiall towards the Center.

In Example II. The Circle of the Sunnes courſe vpon the 24th day of *March*, ſhall bee an imaginary Circle betweene the ſecond, and third Parallels, ſtill keeping an halfe of that ſpace, and one fift part more of the reſt from the ſecond.

In Example III. The Circle of the Sunnes courſe vpon the 13th day of *Nouember*, ſhall be an imaginary Circle, betweene the tenth, and eleauenth Paralels, below the Æquinoctiall, ſtill keeping one quarter of that ſpace from the tenth.

VII. Vſe. *To find the* Riſing *and* Setting of the Sunne *euery day.*

Seeke out (as was laſt ſhewed) the imaginary Circle or Parallel of the Sunnes courſe, for that day, and marke the point where it meeteth with the Horizon, both on the Eaſt and Weſt ſides thereof, for that is the very point of the Sunnes riſing, and ſetting that ſame day, and the hower lines which are on both ſides of it, by proportioning the diſtance reaſonably, according to 15 minuts, for the quarter of the hower, will ſhew the hower of the Sunnes riſing, on the Eaſt ſide, and the Sunnes ſetting on the Weſt ſide.

VIII. Vſe

VIII. Vſe. *To know the reaſon, and manner, of the* Increaſing, *and* Decreaſing of the dayes and nights *throughout the whole yeare.*

When the Sunne is in the Æquinoctiall, it riſeth, and ſetteth at 6 a Clock, for in the inſtrument, the interſection of the Æquinoctiall, and the Ecliptic with the Horizon, is in the 6 a clock Circle on both ſides. But if the Sunne bee out of the Æquinoctiall, declining toward the North, the interſections of the Parallel of the Sunne with the Horizon, is before 6 in the Morning, and after 6 in the Euening: and the diurnall Arch of the Sunne, greater then 12 howers; and ſo much more great, the greater the Northerne Declination is. Againe if the Sun be declining toward the South, the interſections of the Parallel of the Sunne with the Horizon, is after 6 in the Morning, and before 6 in the Euening; and the diurnall Arch leſſer then 12 howers; and by ſo much leſſer, the greater the Southerne Declination is.

And in thoſe places of the Ecliptic in which the Sun moſt ſpeedily changeth his Declination, the length alſo of the day is moſt altered, and where the Ecliptic goeth moſt parallel to the Æquinoctiall changing the Declination but little, the length of the day alſo is but little altered.

As for example, when the Sunne is neere vnto the Æquinoctiall, on both ſides, the dayes increaſe, and alſo decreaſe ſuddenly and apace: becauſe in thoſe places, the Ecliptic inclineth to the Æquinoctiall in a manner like a ſtreight line, making ſenſible declination. Againe when the Sunne is neere his greateſt Declination, as in the height of the Summer, and the depth of Winter, the dayes

dayes keepe for a good time, as it were at one ſtay, becauſe in thoſe places the Ecliptic is in a manner parallel to the Æquinoctiall, ſcarce altering the declination: and becauſe in thoſe two times of the yeare, the Sunne ſtandeth as it were ſtill, at one declination; they are called the *Summer Solſtice*, and the *Winter Solſtice*. And in the meane ſpaces, the neerer every place is to the Æquinoctiall, the greater is the diverſitie of dayes.

Wherefore we may hereby plainely ſee, that the common receiued opinion, that in every moneth, the dayes doe equally increaſe, is erroneous.

Alſo wee may ſee that in Parallels equally diſtant from the Æquinoctiall, the day on the one ſide, is equall to the night on the other ſide.

IX. Vſe. *To find the* Aſcenſionall difference of the Sunne *every day.*

Seeke out the time of the Suns Riſing, or Setting that ſame day (by the VII Vſe) and ſee how much it differeth from ſixe a clocke, then conuert the ſame difference into degrees (as was taught in 1 Part. Chap. 12. Sect. 1.) by multiplying the howers with their decimall parts, by 15. And ſo haue you the Aſcenſionall difference for that day.

X. Vſe. *To find out the* Oblique aſcenſion of the Sunne *every day.*

Seeke out the Sunnes Right aſcenſion (by the IIII Vſe) and the Aſcenſionall difference (by the IX Vſe:) And if the Sunne be in the firſt Semicircle of the Ecliptic, Subduct the Aſcenſionall difference, out of the Right aſcenſion: But if the Sunne be in the ſecond Semicircle of the Ecliptic, adde the Aſcenſionall difference to the Right aſcenſion: and you ſhall haue the oblique aſcenſion.

XI. Vſe. *To find how farre the Sunne riſeth, and ſetteth, from the true Eaſt and Weſt points, which is called the* Sunnes Amplitude ortiue, *and* occaſiue.

Seeke out (as was ſhewed in the VI. Vſe) the imaginary Circle, or Parallel of the Sunnes courſe, and the points of that Circle in the Horizon, on the Eaſt, and Weſt ſides, cutteth the degree of the Amplitude Ortiue, and occaſiue.

XII. Vſe. *To find the* length of every day and night.

Double the hower of the Sunnes ſetting, and you ſhall haue the length of the day; and double the hower of the Sunnes riſing, and you ſhall haue the length of the night.

XIII. Vſe *To find the* true place of the Sunne, vpon the Inſtrument, *which anſwereth to the point, wherein the Sunne is in the heauens: and is the ground of all the queſtions following.*

Take with your Inſtrument the height of the Sunne, and reckon it on the moueable Index, or Labell: and then moue the ſaid Labell, till you find the height of the Sunne, exactly to fall vpon the Parallel of the Sunne for that day, on the Eaſt ſide if it bee in the Fore-noone, and on the Weſt ſide, if it bee in the After-noone; the point of interſection, where the Index, or Labell croſſeth the Parallel

raiel, in that point of the Sunnes altitude, shall bee the true place of the Sunne on the Instrument.

XIIII. Vse. *To find the* Hower of the day.

The true place of the Sunne on the Instrument (found out as was last shewed) sheweth among the hower lines the true hower of the day.

XV. Vse. *To find out the* Azumith *or* verticall Circle *in which the Sunne is, or the Horizontall distance of the Sunne from the Meridian.*

The Index or Labell fastned at the Center, is a moueable Azumith: apply therefore the edge thereof, vnto the true place of the Sunne on the instrument (found out as was shewed by XIII Vse.) And marke what point of the Horizon, or Limbe, the same edge of the Labell cutteth; reckon how many degrees of the Horizon, are intercepted betweene that point, and the Meridian line, or South point, either on the East, or West side: and that Arch shall be the Horizontall distance sought for, whereby is shewed the Azumith of the Sunne at that instant: and consequently the Angle which the verticall Circle, or Azumith of the Sunne maketh with the Meridian.

XVI. Vse. *The* Azumith of the Sunne *being knowne, to find out the* Altitude of the Sunne, *and* the Hower of the day.

Set the edge of the Labell to the Azumith given, and marke in what point the same edge crosseth the Parallel of the Sunne for that day: that point of intersection

ſheweth the height of the Sunne aboue the Horizon, vpon the Labell: and alſo it ſheweth the hower of the day among the hower lines.

XVII. Vſe. *To find at what* hower the Sunne commeth to be *full* Eaſt, *or* Weſt *every day in Summer.*

Apply the edge of the Labell, vnto the Eaſt, or Weſt points of the Limbe, and marke in what point, the ſaid edge cutteth the Parallel of the Sunne for that day, for that ſame point among the hower lines, ſhall ſhew the time of the Sunnes comming to be full Eaſt, or Weſt in that day, and likewiſe of what altitude the Sunne will be aboue the Horizon, at that time of his being full Eaſt, or Weſt.

XVIII. Vſe. *To find the* height of the Sunne at high Noone every day, *and likewiſe at every other hower.*

Marke in what point the Parallel of the Sunne for that day, cutteth the line of that hower, for which you would know the Sunnes altitude: And vnto that point of interſection, apply the edge of the moueable Labell, or Index: and thereon ſhall you find, the very degree of the Sunnes altitude, at that hower.

By this XVIII Vſe, and by the XVI, are made the Quadrants, deſcribed by *Gemma Frizius*, *Munſter*, *Clauius*, Mr. *Gunter*, and others: and alſo all manner of *Rings*, *Cylinders*, & innumerable other *topicall Inſtruments*, for the finding out of the hower, and other like concluſions. And likewiſe the reaſon, of finding the hower of the day, by a mans ſhadow, or by the ſhadow of any Gnomon, ſet vp perpendicular to the Horizon, or elſe parallel to it.

XIX. Vſe,

XIX. Vſe. *To find out the* Meridian line, and the points of the compaſſe *without a Magneticall needle, yea more exactly then with a needle.*

Take the height of the Sunne, by the ſhadow of the pinne: and apply the ſame height, reckoned on the Index, or Labell to the parallel of the Sunne for that day, whereby you haue the true place of the Sunne, in the inſtrument, as hath beene ſhewed in the XIII. Vſe. Then keeping both the Labell at that point, and the pinne vpright in the Center, hold, or ſet your inſtrument parallel to the plaine of the Horizon, with the pinne toward the Sunne, and moue it gently, till the ſhadow of the pinne ſhall fall, exactly vpon the fiduciall edge of the Labell. For then the Meridian line of the inſtrument, ſhall be in the true Meridian of the place: and the foure quarters of the inſtrument, ſhall looke into the foure cardinal points, of Eaſt, Weſt, North, and South. Wherefore if with a bodkin, you make a prick at each end of the Meridian of your inſtrument where it ſtandeth: and with a Ruler draw a line through them: the ſame ſhall bee the Meridian of that place.

This is a moſt excellent practiſe, for finding out the Meridian in any place, and is in an inſtant performed, and that eaſily. And hereby you may examine the *Variation of the Compaſſe*. And alſo exactly place any Sunne Dyall.

XX. Vſe. *Conſiderations for the* vſe of the inſtrument in the night.

In ſuch queſtions as concerne the night, or the time before Sun riſing, and after Sunne ſetting, the inſtrument repreſenteth the lower Hemiſphære, wherin the Southerne

Pole is eleuated. And therefore the Parallels, which are aboue the Æquinoctiall, ſhall bee for the Southerne, or Winter Parallels, and thoſe beneath the Æquinoctiall, for the Northerne, or Summer parallels. And the Eaſt ſhall be accounted for Weſt, and the Weſt for Eaſt: and the North ſhall bee accounted for South, and the South for North: contrary to that which was before, when the Inſtrument repreſented the vpper Hemiſphære.

XXI. Vſe. *To find* how many degrees the Sunne is vnder the Horizon, at any time of the night.

Seeke the declination of the Sunne for the day propoſed: and at the ſame declination, on the contrary ſide, imagine a Parallel for the Sunne that night: and marke what point of it is in the very hower and minute propoſed: then ſet the Index, or Labell to that point of the Parallel, and it will ſhew you thereon the degree of the Sunnes depreſſion vnder the Horizon.

XXII. Vſe. *To find out the* length of the Crepuſculum, or Twilight *euery day.*

Becauſe the queſtion concerneth the night time, you muſt ſeeke out the Sunnes Parallel, for the night, on the other ſide of the Æquinoctiall, hauing the ſame declination with that which the day of the moneth ſheweth: then moue about the Labell, vntill the ſaid Parallel cutteth the edge thereof in the 18th deg: on the Weſt ſide for the Morning Twilight, and on the Eaſt ſide for the Euening Twilight, of the ſame day.

And note that in the height of Summer, the Twilight in our Horizon, continueth all night long: becauſe the

the same goeth not vnder the Horizon, full 18 degrees

XXIII. Vse. *To find the* Declination of any Wall, or Plaine.

Take a board hauing one streight edge, and a line drawne perpendicular vnto that edge: apply the streight edge vnto the Wall, at what time the Sunne shineth theron, holding the board parallel to the plaine of the Horizon: and hang vp a thread with a plummet, so that the shadow of the thread may fall on the board, crossing that perpendicular line. Then take with your Instrument the height of the Sunne, and instantly make two pricks, in the shadow of the thread on the board, a good way distant one from the other: and laying a Ruler to those two pricks, draw a line, which line shall be the Azumith of the Sunne, on the board: againe with the height of the Sunne lastly taken, find out on your instrument, the Azumith of the Sunne; or the Angle which the Sunnes Azumith maketh with the Meridian, (by the XV. Vse.) And on the board taking the intersection of the shadow line with the perpendicular for the Center, describe a Circle equall to the innermost Circle of the Limbe: (which you may easily doe, if you set one foot of your compasses vpon the East, or West point, and extend the other foot vnto 60 degrees, on the same innermost Circle, for this distance is equall to the *Radius* thereof.) Againe with your compasses, take of the Arch betweene the Azumith of your Instrument, and the Meridian, and set that on the Circle of the board, that way that the true South is: and through the end of that Arch measured on the board, draw a streight line for the Meridian. Lastly take with your compasses, the Arch intercepted between the Meridian on the board, and the perpendicular line, and by applying it to the inmost Circle of the limbe, from the East, or West points see how many degrees it con-

containeth: for that is the declination of the Wall. Or else you may find the Meridian vpon the board, by X.X Vſe.

If the Angle of the Meridian with the perpendicular, on the board, be a right Angle, the Wall is direct East, or West.

But if the Meridian fall vpon the perpendicular, or be parallel thereto making no Angle with it, the Wall is direct North, or South.

XXIIII. Vſe. *The* Art of Dyalling.

And firſt how to make the Inſtrument in paper, promiſed in the beginning of this ſecond part.

For the Delineation of this inſtrument in paper, it will bee neceſſary firſt to ſhew the manner how the Semidiameter is to bee graduated, or diuided into degrees: and how the Centers, and Semidiameters, of the ſeuerall kinds of Arches are to be found.

Vpon hilte a ſheet of ſtrong large Dutch paper, (the larger, the better) draw two ſtreight lines, making a right Angle neere one of the corners, the one through the length, and the other through the breadth of the paper; which two lines I therefore call the longer, and the ſhorter perpendicular.

Vpon the right Angle point, being the Center, with a Semidiameter equall to that by which you intend to delineate your inſtrument, deſcribe a quadrant of a Circle: and on the point where it meeteth with the ſhorter perpendicular, draw a long tangent line parallel to the longer pependicular.

Diuide the Quadrant into 90 degrees, among which from the beginning at the ſhorter perpendicular, reckon the eleuation of the Pole, for which you will make your in-

inſtrument, and applying a Ruler to the end thereof, and to the Center, where the Ruler cutteth the tangent line make a prick. And taking with your compaſſes the diſtance from the Center to that prick, meaſure it vpon the ſhorter perpendicular: this ſhall be the *Semidiameter of the ſixt hower Circle.* At the end thereof draw another long line parallel alſo to the longer perpendicular.

Then out of the Center vnto the ſecond parallel through every degree of the quadrant, draw fine ſtreight lines, cutting alſo the firſt Parallel. The interſection of thoſe lines with the firſt Parallel, ſhall be *The ſcale of centers of Arches.* And their interſection with the ſecond Parallel ſhall be *The ſcale of centers of hower Circles.* And the ſegments of thoſe lines, intercepted betweene the Center, and the firſt Parallel, ſhall be the *Semidiameters of Arches*: and the whole lines betweene the Center and the ſecond Parallel, ſhall bee *The Semidiameters of hower Circles.* And that you may know for what Circle, every Center, and Semidiameter ſerueth, you ſhall note every fift line from the beginning, with the figures 5, 10, 15, 20, &c. Set vnder the ſecond Parallel, vnto 90 which will fall vpon the longer perpendicular: that ſo you may readily find the Center, and Semidiameter of any Circle required.

Againe divide the firſt 45 degrees of the Quadrant in the middeſt: and applying your Ruler to the Center, and to every one of thoſe halfe diviſions, where in each place the Ruler cutteth the firſt Parallel, or tangent line, make a prick. So ſhall you haue vpon the tangent line betweene the ſhorter perpendicular and the midlemoſt line 45, a third ſcale, which is, *The ſcale of 90 degrees*, for the graduating of the Semidiameter of your inſtrument on the paper: In which you ſhall alſo diſtinguiſh every fift degree, with figures ſet vnder the tangent line.

Having thus prepared your paper of ſcales with lines

neatly and exactly drawne, keepe it by you to haue it ſtill in a readineſſe for the making, and vſing of the Inſtrument in paper. The making whereof is thus.

Take with your compaſſes the Semidiameter of the Quadrant in your paper of ſcales: and therewith vpon a peice of ſtrong Dutch paper, Deſcribe the Horizontall Circle: which you ſhall cut into two Semicircles with a Meridian line drawne through the Center: diuide them into Quadrants in the points of East, and West: and each Quadrant into 90 degrees to be marked with figures juſt as is done in the Inſtrument.

Then with your compaſſes take the elevation of the pole vpon the ſcale of degrees in your paper: & ſet it vpon the Meridian line from the Center which way you pleaſe: that ſhall bee the interſection of the Æquinoctiall with the Meridian. Alſo reckon the complement of the height of the Pole, vpon the ſcale of Centers of Arches, and with your compaſſes take the diſtance from the end thereof to the Center: the ſame ſhall bee the Semidiameter of the Æquinoctiall, to bee drawne from the East point of the Horizon through the point of interſection with the Meridian vnto the Weſt point.

Againe take with your compaſſes vpon the ſcale of degrees in your paper the complement of the height of the Pole: and ſet it vpon the Meridian on the other ſide of the Center from the Æquinoctiall: there ſhall bee the Pole of the Æquinoctiall, or of the World, in which all the houre lines ſhall croſſe one another.

Nextly vnto the height of the Pole both adde, and alſo ſubduct 23 ½ degrees: and with your compaſſes take both theſe Arches in the ſcale of degrees: and ſet them in the Meridian from the Center, one falling beyond the Æquinoctiall, and the other ſhort of it: thoſe ſhall be the interſections of the two Tropics with the Meridian. Seek

alſo

and by Sect: 8, Chapt: 12 of the firſt part, the Amplitude ortiue of the Sun, having 23½ degrees declination: and reckon it being found, in the Horizontall Circle from the Eaſt and Weſt points both wayes: thoſe are the points of interſection of the Tropics with the Horizon on both ſides: and ſo having three points in each Tropic, you may eaſily through them drawe the Circles.

Moreover with your compaſſes, take the diſtance betweene the Center and the ſecond Parallel in your paper, which is the Semidiameter of the ſixt houre Circle: and ſet it on the Meridian from the Pole beyond the Æquinoctiall: that ſhall be the Center of the ſixt houre Circle: vpon which you may draw the ſame Circle, from the Eaſt point of the Horizon through the Pole to the Weſt point. Then through the center of the ſixt houre Circle erect a line perpendicular to the Meridian, extending it infinitly on both ſides of the Meridian: and in that line both wayes, pricke downe the Centers of the horary Circles, out of the ſcale in paper: And laſtly opening your compaſſes from every one of thoſe Centers vnto the Pole ſeverally, deſcribe all the horary Circles, or at leaſt every fift of them, and ſo is your paper inſtrument perfectly finiſhed.

The vſe of this inſtrument on paper is, that lines, and arches may bee deſigned vpon it with a fine pennicell of blacke lead, and afterward be wiped out againe. Wherefore it will bee needfull for him that will vſe this inſtrument, to all the purpoſes thereof, to get a good paire of large compaſſes with three points, one ſharpe, another for Inke, a third for blacke Lead. And I ſuppoſe it would doe well to faſten over your inſtrument a peice of thinne oyled paper, through which the lineaments may be conſpicuous: and vpon it to trace ſuch lines, and arches as you haue occaſion to vſe: that ſo your inſtrument may be kept cleane, and laſt longer.

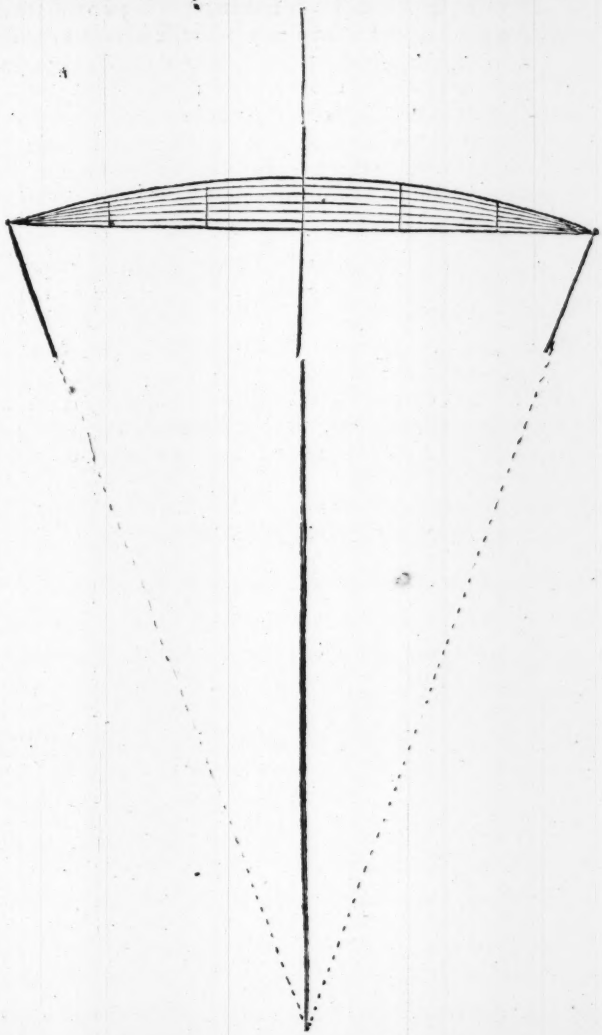
For as much as in delineating the horary Circles, which are within 30 degrees of the Meridian, the Semidiameters will be too long for your compasses: you may in that streight thus helpe your selfe. First say,

As the *Radius*,
is to the Sine of the *Elevation of the Pole*;
So is the tang: of the *distance of any Horary Circle from the Meridian*, suppose 25,
or 20, or 15, or 10, or 5 degrees,
to the *arch of the Horizon betweene the Meridian, and that horary Circle.*

Reckon this distance on the limbe of your Instrument from both ends of the Meridian, and marke it. Thus doe for the 25th, 20th, 15th, 10th, and 5th horary Circle on both sides of each end of the Meridian.

Then in any peice of cleane paper, through the middest of the longer way, draw a line: and toward one end which (I call the upper ende) crosse the same with a perpendicular line exactly equall to the Diameter of your Instrument, the point of Intersection being the center.

Take with your Compasses out of the paper of scales, the semidiameter of 60. degr: (which you may well doe for an ordinary instrument): and setting one foot on either end of the Diameter, that point wherein the other foot shall cutt the first long line, make your Center, and thereon draw an Arch through both ends of the Diameter, and cutting the vpper part of the first long line: this Arch is equall to that horary Circle, which is distant from the Meridian 30 degrees the complement of 60.



Divide each halfe of the Diameter into 3 equall parts, with 4 points, and from every of thoſe points vnto the Arch draw lines paralel to the firſt long line. And having divided euery one of thoſe fiue paralel lines intercepted betweene the Diameter and the Arch into 6 equall parts, for the 6 times 5 degrees which remaine to the Meridian, draw through thoſe diviſions from the ends of the Diameter (with a ſmooth and even hand) the Arches 25 20, 15, 10 and 5.

Thoſe Arches you may transferre from the paper to your inſtrument in this manner. Rubb the backe ſide of the paper againſt the Arches, with fine powder of blacke lead: then applying the paper with Arches to your inſtrument, that the ends of the Diameter may exactly fall vpon the two oppoſite markes, in the limbe of your inſtrument, which ſerue for the horary Circle that you would draw, either 25, 20 15 10, or 5, trace over that Arch with the point of any hard peice of wood ſharpened: and the blacke lead on the backe ſide will vpon the inſtrument leaue the print of that Arch.

XXV. Vſe. *To ſet an vpright Wall or plaine vpon the inſtrument: and to find how many houres the Sunne ſhall ſhine thereon at ſome time of the yeare.*

The ſituation of Wals, or Plaines is conſidered either in reſpect of the Meridian, or of the Horizon. And vnto both it is either perpendicular, or oblique, or parallel.

The plaine perpendicular to the Meridian, is that which ſtandeth directly North, or South: which if it be alſo perpendicular to the Horizon, is called North, or South direct vpright. But if it ſloope from the *Zenith* forward, it is called North, or South inclining: If backeward

ward, it is called North or South-reclining. And note that in a ſtooping Plaine that ſide which is toward the Horizon, is inclining, and that which is toward the Zenith is reclining.

The *Plaine oblique to the Meridian* is that which ſtandeth not directly North, or South, but declineth one ſide into the Eaſt, and the other into the Weſt: and is therefore called *Declining Eaſtward, or Weſtward*, according as either ſide of the Plaine looketh: As if an vpright Wall being Southerne, decline from the South into the Eaſt, it is called *South declining Eaſtwards vpright*. But if it be not vpright, it is called *South declining Eaſtward*, and *inclining, or reclining*.

The *Plaine parallel to the Meridian*, is that which looketh directly Eaſt, or Weſt; and accordingly, hath his denomination, whether it bee *Vpright, Inclining, or Reclining*.

The *Plaine Parallel to the Horizon*, is called Horizontall: and is repreſented by the inſtrument it ſelfe, or at leaſt by the innermoſt Circle of the limbe thereof.

And note that the Arch of Declination, is reckoned from the next Eaſt, or Weſt point. And that the Arch of Inclination, or Rectination is reckoned from the Zenith, or the complement of it from the Horizon. So that euery vpright Plaine is vnderſtood to paſſe through the Zenith, which in the inſtrument is the Center.

And thus having ſhewed the ſeuerall affections of Plaines, wee will now proceed to ſhew the manner how to ſet them vpon the Inſtrument.

A Direct North, or South vpright Plaine, is repreſented in the Inſtrument by a line drawne through the Center from the Eaſt point to the Weſt, which is alſo the Horizontall interſection of the Plaine. And by it you ſhall

ſhall ſee that the Southerne ſide or face of the plaine is open to all the houres betweene ſixe in the morning and ſixe in the evening. And that about *London*, the Northerne ſide, onely in the Summer enioyeth the Sunne from his riſing till after ſeven in the morning: and from before 5 a clocke in the afternoone, till his ſetting.

A direct Eaſt, or Weſt upright plaine, is repreſented in the Inſtrument by the Meridian, which is alſo the Horizontall interſection of the plaine. And in it you ſhall ſee that all the forenoone houres are open to the Eaſt ſide: and all the afternoone houres to the Weſt ſide.

A Declining Plaine is thus ſet upon the Inſtrument, reckon on the Horizon the arch of Declination, from the Eaſt, or Weſt point: and at the end draw a line through the Center vnto the oppoſite point of the Horizon: So that each ſide thereof may be open to that point, either Eaſt, or Weſt, into which the Declination is ſuppoſed. That line ſo drawne throught the center is the Horizontall interſection of the plaine, and repreſenteth the plaine it ſelfe, if it bee vpright. For example, there is about *London* an vpright Wall declining Eaſtwards 35 degrees: which I would ſet vpon the Inſtrument. Hold the Southerne part of the Inſtrument to you, and reckon from the Eaſt backward into the North vpon the Horizon 35 degrees: there draw a line through the Center: this line ſhall not onely vpon the South ſide repreſent a Southerne Plaine declining Eaſtward 35 degrees. But alſo vpon the North ſide ſhall repreſent a Northerne Plaine declining Weſtward 35 deg. And moreover it will appeare that on the Southerne ſide ſhall bee drawne the houres from almoſt 4 a clocke in the morning, till 3 in the afternoone. And that in the Northerne ſide ſhall bee drawne vpon one ſide 4 a clocke in the morning onely: and vpon the other ſide all the houres from 3 in the afternoone till Sunne ſet.

And

And to conſequently the declination of an vpright wall, or Window being given, it may be found at what houre the Sunne vpon any day in the yeare will come to that Wall, or Window, and when it will goe from it. As in the former example, There is about *London* a Northerne wall declining Weſtward 35 deg. I would know at what time of the day the Sunne will begin to ſhine vpon it on the 24th day of *March* Set the Index at 35 deg: from Weſt toward South: and becauſe that day the Sunnes Declination is 6 degrees Northward; Looke at what houre the ſixt Parallel aboue the Æquinoctiall toward the Center meeteth with the Index ſo placed: and you ſhall find it at 3 a clock in the Afternoone. Wherefore at that time the Sunne will begin to ſhine vpon the Wall that ſame day.

The Poles of every vpright Wall are in the Horizon 90 deg: that is a quarter of a Circle, diſtant from the line repreſenting the Plaine. Wherefore if vpon that line in the Center you erect a perpendicular, the ends therof in the Horizon ſhall be the poles of that Plaine: and are ſo farre diſtant from the North and South points, as the Plaine it ſelfe is from the Eaſt, and Weſt.

XXVI. Vſe. *To ſet an* Inclining, *and* Reclining Wall, *or* Plaine *vpon the Inſtrument: and to find* how many houres the Sunne ſhall ſhine thereon, *at ſome time of the yeare.*

When you haue *an Inclining*, or *Reclining Plaine* to be deſcribed on the Inſtrument. Firſt the Horizontall interſection is to be ſet thereon, as if it were vpright; together with the line perpendicular thereto, in which are the Poles of the Plaine: according as was taught in the XXV Vſe.

Then vpon the ſcale of degrees in your paper, reckon the arch of Inclination, or Reclination; and with your compaſſes take, & ſet it in your Inſtrument vpon the line perpendicular to the Horizontall interſection of your Plaine, from the Center that way into which the Inclination, or Reclination tendeth: the ſame ſhall bee the vppermoſt point of your Plaine.

Againe, with your Compaſſes take the Complement of inclination, or reclination, both upon the ſcale of degrees, and alſo upon the ſcale of centers of arches in your paper: and ſet both ſpaces upon the ſame perpendicular line, but on the other ſide of the center (extended if need be): At the ſhorter of thoſe ſpaces ſhal be the pole of your plaine: and at the longer of them ſhal be the Center of it.

Laſtly, ſetting one foot of your Compaſſes in the center of your Plaine, and extending the other foot to the vppermoſt point, deſcribe in your Inſtrument an Arch of a Circle: which if you haue done well, will exactly fall vpon the ends of the Horizontal interſection of your Plaine. That Arch ſhall repreſent your Plaine, inclining vpon the lower ſide, which is toward the Horizon, or Limbe: but reclining vpon the vpper-ſide, which is toward the Zenith, or Center. And to either ſide ſhall ſhew in what hower lines the Sunne, at ſome time of the yeare, will ſhine vpon it: that in delineating a Dyall thereon, it may not be combered with vnneceſſary houre lines. For Example, ſuppoſe that the former Plaine, which with the South declined Eaſtward 35 deg: doe alſo incline 41 deg: 30 min. Wherefore alſo with the North ſide it ſhall decline Weſtward 35 deg: and Recline 41 deg: 30 min. Deſcribe this plaine vpon your Inſtrument with an Arch of a Circle, found out as was taught laſt before. And it will appeare that vpon the Inclining ſide ſhall be drawne all the houre lines from almoſt 4 in the morning, to 4 in the afternoone: And vpon the Reclining ſide ſhall bee drawne firſt 4 and 5 in the morning: and then beginning at 9 a clocke, all houres to Sunne ſet.

And

And note that in all Northerne plaines, the North Pole is eleuated: and in all Southerne plaines the South Pole is eleuated. Except such North inclining, and South reclining Plaines, that in the Instrument fall below the North Pole, betweene it and the Horizon: For in them the contrary Pole is eleuated. And also that a direct East and West plaine, if it Recline, hath the North Pole eleuated: and if it Incline, the South Pole.

XXVII. Vse. *The Plaine being set vpon the Instrument, to find the* distances of the houre lines, *and* Substile from the Horizontall Intersection. *And also the* height of the Stile aboue the Substile.

Euery Dyall either hath a Center in which all the houre lines, together with the Substile, and Stile doe meet: or else it hath no center, & so they are al parallel one to another.

If the Plaine being set vpon the Instrument, cutteth the Pole of the Æquinoctiall (that is the point in which all the horary Circles crosse the Meridian) the Dyal to be drawne vpon that Plaine shall haue no Center. But if it cutteth not the Pole, the Dyal shall haue a Center. And of these Dyals with Centers wee will first intreate: as being most proper for the vse of the Instrument.

Behold therefore the Pole of your plaine heedily what horary Circle it falleth vpon: Or if it fall betweene any two, the distance of each being reasonably apportioned, imagine a horary Circle passing through it. Marke in what point that horary Circle, either real, or imagined, doth cut the Plaine, that same point shall bee the place of the Substile in your Plaine: and the height of the Stile aboue it, is the Arch of that horary Circle intercepted betwixt the Pole of the Æquinoctial, and the point of the Substile noted in the Plaine.

Therefore applying a Ruler to the Pole of your Plaine, carry it about vnto all the interſections of the Plaine with the houre Circles, and the ſubſtile ſeuerally: and where in euery place the Ruler ſhall cut the innermoſt Circle of the Limbe, there make a viſible marke: For the arches of the Limbe intercepted betweene the Horizontall points of your Plaine, and euery one of thoſe marks, ſhall be the diſtance intended to be ſought.

But for the Horizontall plaine, the ends of the houre Circles in the Limbe of your inſtrument, doe giue the diſtance without any more adoe.

Concerning the height of the Stile aboue the Subſtile: It is apparent by the inſtrument, that in a Horizontal dyal, the 6 a clocke line lyeth directly Eaſt and Weſt: and the Meridian perpendicular to it, directly North and South. And that the Meridian is the Subſtile. And that the height of the Stile aboue the Subſtile, is equall to the height of the Pole in that place.

It is alſo apparent, that in all direct North and South Dyals, the 6 a clocke Line is drawne parallel to the Horizon, and the Meridian perpendicular to it: And that the Meridian is the ſubſtile. And that the height of the Stile aboue the ſubſtile, if the Plaine bee vpright, is equall to the complement of the height of the Pole. But if it be North inclining, or South Reclining, it is equall to the difference, of the height of the Pole, and the Arch of Inclination, or Reclination. And if the Plaine bee North Reclining, or South Inclining, it is equall to the Summe of the complement of the height of the Pole and of the Arch of Inclination, or Reclination. And if the Plaine fall vpon the Æquinoctiall, the ſtile ſhall ſtand vp perpendicular vpon it in the Center: and the houer lines ſhall be drawne all at equall Angles, *viz.* 15 degrees one from another.

In

In ſuch Dyals as haue not the Meridian for the ſubſtile, the height of the Stile aboue the Subſtile is thus found by the Inſtrument. It was ſhewed before that the height of the ſtile aboue the ſubſtile is the Arch of the horary Circle through the Pole of the Plaine, intercepted betweene the pole of the Æquinoctiall, and the point noted in the Plaine for the ſubſtile. Therefore from the Horizontall points, or interſections of that horary Circle reckon 90 degrees both wayes: and thereto through the Center draw an obſcure line: in which line ſhall be both the Inclination of that horary Circle, which is the diſtance of the interſection, or vppermoſt point thereof from the Center: and alſo the Pole. Then with your compaſſes take that diſtance, or Inclination: and ſetting it vpon the ſcale of degrees in your paper, ſee how many degrees it containeth vpon that ſcale. Againe vpon the ſame ſcale take with your compaſſes the complement of that inclination or diſtance, which being ſet vpon the obſcure line on the other ſide of the Center, ſhall ſhew the Pole of that horary Circle. Laſtly applying a Ruler to the Pole of that horary Circle, and both to the Pole of the Æquinoctiall, and to the point of the ſubſtile in the Plaine ſeuerally: marke where in both places the Ruler cutteth the innermoſt limbe of your inſtrument: For the degrees of the limbe intercepted betwixt thoſe markes, ſhall be the height of the ſtile, aboue the ſubſtile, which was ſought for.

And by this which hath beene taught, you ſhall find that in an vpright Dyall declining, as before 25 deg: from the South into the Eaſt, or from the North into the Weſt, the ſubſtile ſhall fall vpon that horary Circle, which is about 3 deg: after 9 a clock in the morning: and the ſtile elenated aboue the ſubſtile about 31 deg: And alſo that in a South Dyall declining Eaſtward 35 deg: and inclining 41 deg: 30 min: Or in a North declining Weſtward 35 deg: and reclining 41 deg: 30 min: the

ſubſtile ſhall fall vpon that horary Circle which is about 8 degrees after 6 a clock: and the ſtile eleuated aboue the ſubſtile about 63 deg. 30 minutes.

XXVIII. Vſe. *The making of all manner of plaine* Dyals with Centers.

I haue already ſhewed how to find out in our Inſtrument the diſtances of all the houre lines, and the ſubſtile, from the Horizontal interſection. Now the delineating of a Dyal is nothing elſe but to tranſferre thoſe diſtances out of your Inſtrument into the Dyal plaine, euery one in his due ſituation: and then through them, out of the Center, to draw ſuch houre lines as ſhall be of vſe, together with the ſubſtile.

The due ſituation of thoſe diſtances vpon the Dyal plaine, dependeth on the true placing of the Meridian, or 12 a clocke line: for that being truely deſcribed, all the reſt will be eaſie enough.

Firſt therefore I will ſhew the manner how the Meridian, or 12 a clocke line, is to be deſcribed.

Take in your Dyal ſome point for the Center, where you ſhall thinke fit: and through it draw a line parallel to the plaine of the Horizon. Croſſe it in the Center with a perpendicular line. And hauing opened your compaſſes to the length of the Semidiameter of your paper Inſtrument, deſcribe on the Center a Circle equall to the innermoſt Limbe thereof. In which Circle the line parallel to the Horizon is for the Horizontal interſection: and the other for the line perpendicular to it: and the Circle it ſelfe repreſenteth the plaine: Marke therein the Eaſt and Weſt ſides of the Plaine with *E* and *W*.

In the Horizontal, and in all North and South direct Plaines, both vpright, and ſtooping; and in all vpright decli-

declining plaines, the Meridian is perpendicular to the Line parallel to the Horizon.

In North inclining, and South reclining plaines, the Meridian is to bee drawne on that ſide of the Dyal plaine either Eaſt, or Weſt, into which the declination is: But in North inclining and South reclining, on the contrary ſide. And if the plaine bee Northerne, the Meridian ſhall be aboue the Line parallel to the Horizon: and if the plaine be Southerne, it ſhal be vnder it. And if the contrary Pole be eleuated, it ſhall be drawne through the Center into the oppoſite Quadrant of the Circle in your Dyal plaine.

Laſtly in a direct Eaſt and Weſt plaine, both inclining and reclining, the Meridian is the ſame with the line parallel to the Horizon.

Wherefore with your compaſſes take the diſtance in the limbe of your Inſtrument, from the next Horizontall point, vnto the marke of the Meridian; and meaſure it vpon the Circle of the Dyal plaine, in that part, and on that ſide, according as in conſideration of the eleuated Pole, and of the qualitie of the Plaine, was ſhewed to be agreeable. And at the end of that arch, through the Center, draw a line for the Meridian.

Againe with your compaſſes take the diſtances in the limbe of your Inſtrument, betweene the marke of the Meridian, and the markes of all the houre Lines ſeuerally: and ſetting them vpon the Circle of the Dyal plaine orderly from the Meridian, the Forenoone houres on the Weſt ſide of it, and the Afternoone houres on the Eaſt ſide: at the end of euery one of thoſe arches draw the houre Lines: and diſtinguiſh them with their proper figures accordingly.

Laſtly faſten the ſtile in the Center, ſo that it may hang perpendicular vnto the plaine in the Subſtile, at the iuſt height. And becauſe the ſtile in every Dyal is vnderſtood to be a ſegment of the Axis of the world

which

which is a line imagined to passe from the North to the South Pole through the Center of the earth; the stile being rightly placed shal stil with the end point towards the eleuated Pole, that is vpward from the Center, if the North Pole be eleuated; or downeward from the Center if the South Pole be eleuated.

XXIX. Vse. *The making of all manner of plaine* Dyals not hauing Centers.

If the plaine represented on the Instrument (as was taught before in the XXV and XXVI Vses) cut the Pole of the Æquinoctiall, it is an horary Circle, either one of them which are drawne in the Instrument, or falling betweene some two of them: and the Dyall plaine it selfe shall not crosse the axis of the world, but lye parallel to it, without any Angle of eleuation. And therefore such a Dyal can haue no Center: But the stile, the substile, and all the houre lines shall be parallel one to another.

Every such Plaine represented on the Instrument,

Either, *First* it is the Meridian of the place, the Horizontall intersection whereof is the 12 a clocke Line drawne from North to South: and the Dyall made thereon, is a direct East, or West vpright Dyal: In which the substile is distant from the Line, in the Circle of the Dyall plaine parallel to the Horizon, with an Arch equall to the eleuation of the Pole, and vpward toward the Pole. And is also the 6 a clocke line in your Dyal.

The rest of the houre lines are thus described. Draw through the substile, in any point, a long Line at right Angles: that line shall bee the Æquinoctial intersection vsually called the *Contingent line*: And taking a conuenient distance for the stile to hang parallel over the substile (according to the greatnesse of your Dyall plaine) measure

measure it vpon the substile from the Æquinoctiall intersection: and vpon the end of that measure, describe halfe a Circle for the Æquinoctiall it selfe. Diuide each Quadrant thereof from the substile, into 6 equall parts, or houres. Then applying a Ruler to the Center, and to euery one of those diuisions seuerally, where in euery place the Ruler shall cut the long line of Æquinoctiall intersection, make pricks: and through those pricks draw the houre lines, parallel to the Substile, or 6 a clocke line: distinguishing so many of them as bee needfull, with their figures: that is all the Forenoone houres on the East plaine, and all the Afternoone houres on the West plaine. But in these Dyals there is no 12 a clock line, it being infinitly distant from the Substile. Lastly hang the stile directly over the Substile, and parallel to it, at the distance formerly taken. And thus are your East, and West Dyals finished.

Or *Secondly*, it is the sixt houre Circle, the Horizontal intersection whereof is the line of East, and West: and the Dyal made thereon is direct North inclining, or South reclining, with an Arch equal to the complement of the height of the Pole. And the parallel to the Horizon is the Æquinoctial intersection: and the line perpendicular to it is the 12 a clocke line, and also the Substile.

The rest of the houre Lines, from 7 a clocke in the morning, to 5 in the euening, are thus described. Take a conuenient distance for the Stile from the Substile, measuring it vpon the Substile from the Æquinoctial intersection: and on the end of that space describe the Semicircle of the Æquinoctial, to bee diuided on both sides of the Substile into 6 houres: through euery one of which out of the Center, a Ruler being applyed; at the points of the seuerall intersections of the Ruler with the Æquinoctial intersection, draw the houre Lines parallel to the Substile, or 12 a clocke Line: distinguishing them with their figures, namely 11, 10, 9, 8, 7, on the West side:

and 1, 2, 3, 4, 5, on the East ſide: but in theſe Dyals there is no ſixe a clocke Line, it being infinitly diſtant from the Subſtile. Laſtly hang the Stile directly over the Subſtile, and parallel to it, at the diſtance formerly taken.

Or *Thirdly*, it is North inclining, or South reclining, and alſo declining: in which.

As the tangent of the *Elevation of the Pole*,
is to the *Radius*;
So is the Sine of the compl: of *Declination*,
to the tang: of the comp: of *Inclination* or *Reclination*.

The Plaine being ſet vpon the Inſtrument by the Arches of Declination and ſtooping thereof (as hath beene taught in XXVI Vic) ſhall cut the pole of the Æquinoctiall. Apply therefore a Ruler to the Pole of the plaine, and to the Pole of the Æquinoctial; and the point in which it cutteth the Limbe, marke for the ſubſtile: which is to bee transferred vnto the Circle of the Dyal plaine, by taking the diſtance betweene that point, and the next Horizontal interſection, and ſetting it on that Circle from the line parallel to the Horizon, vpward if the plaine be North: or downe-ward if the plaine bee South: and on that ſide which is contrary to the Declination. The ſubſtile being thus found, draw a long line perpendicular to it, for the Æquinoctial interſection. And taking a conuenient diſtance for the ſtile from the ſubſtile, meaſure it vpon the ſubſtile from the Æquinoctiall interſection: and on the end of that ſpace deſcribe the Semicircle of the Æquinoctial. Then looke in your Inſtrument how many degrees of the Æquinoctial are intercepted betweene the Meridian, and the Arch repreſenting your Plaine: and reckoning the ſame number of degrees vpon the Æquinoctial of the Dyal plaine, from the ſubſtile towards the ſide of Declination, there make a marke for the Meridian point thereof: in which

you

you muſt begin to diuide the Æquinoctial ſemicircle into houres both wayes: And that being diuided, apply a ruler to the center, and to euery one of the diuiſions: and at the points of the ſeuerall interſections of the ruler with the Æquinoctial interſection, draw the houre lines parallel to the ſubſtile. Set 12 at that houre line which was drawne at the interſection through the Meridian point of the Equinoctial: and 11, 10, 9, 8, &c. on the Weſt ſide: and 1, 2, 3, 4, &c, on the Eaſt ſide. Laſtly, hang the ſtile directly ouer the ſubſtile, and parallel to it, at the diſtance formerly taken.

XXX Vſe. *How by* Sines and tangents *to calculate the* places of the Meridian, *and* Subſtile, *and the* height of the Stile *aboue it: and the* diſtance of the Meridian of the Æquinoctiall from the Subſtile; *together with the* places of houre lines, *both by calculation, and alſo Geometrically.*

I haue already taught the making of all manner of plaine Dyals moſt eaſily by the Inſtrument, for the ſame height of the Pole. But if any man either want an Inſtrument, or elſe deſireth greater exactneſſe, I will alſo here ſhew how to performe the ſame by calculation, on the other ſide of the Inſtrument.

In a plaine erect Dyall declining.

As the *Radius*
is to the Sine of *declination*;
So is the tang. of the compl. of the *Poles height*.
to the tang. of the *diſtance of the*
ſubſtile from the Meridian.

and 1, 2, 3, 4, 5, on the East ſide: but in theſe Dyals there is no fixe a clocke Line, it being infinitly diſtant from the Subſtile. Laſtly hang the Stile directly over the Subſtile, and parallel to it, at the diſtance formerly taken.

Or *Thirdly*, it is North inclining, or South reclining, and alſo declining: in which.

As the tangent of the *Elevation of the Pole*,
is to the *Radius*;
So is the Sine of the compl: of *Declination*,
to the tang: of the compl: of *Inclination* or *Reclination*.

The Plaine being ſet vpon the Inſtrument by the Arches of Declination and ſtooping thereof (as hath beene taught in XXVI Vic) ſhall cut the pole of the Æquinoctiall. Apply therefore a Ruler to the Pole of the plaine, and to the Pole of the Æquinoctial; and the point in which it cutteth the Limbe, marke for the ſubſtile: which is to bee transferred vnto the Circle of the Dyal plaine, by taking the diſtance betweene that point, and the next Horizontal interſection, and ſetting it on that Circle from the line parallel to the Horizon, vpward if the plaine be North: or downe-ward if the plaine bee South: and on that ſide which is contrary to the Declination. The ſubſtile being thus found, draw a long line perpendicular to it, for the Æquinoctial interſection. And taking a conuenient diſtance for the ſtile from the ſubſtile, meaſure it vpon the ſubſtile from the Æquinoctiall interſection: and on the end of that ſpace deſcribe the Semicircle of the Æquinoctial. Then looke in your Inſtrument how many degrees of the Æquinoctial are intercepted betweene the Meridian, and the Arch repreſenting your Plaine: and reckoning the ſame number of degrees vpon the Æquinoctial of the Dyal plaine, from the ſubſtile towards the ſide of Declination, there make a marke for the Meridian point thereof: in which

you

you muſt begin to diuide the Æquinoctial ſemicircle into houres both wayes: And that being diuided, apply a ruler to the center, and to euery one of the diuiſions: and at the points of the ſeuerall interſections of the ruler with the Æquinoctial interſection, draw the houre lines parallel to the ſubſtile. Set 12 at that houre line which was drawne at the interſection through the Meridian point of the Equinoctial: and 11, 10, 9, 8, &c. on the Weſt ſide: and 1, 2, 3, 4, &c, on the Eaſt ſide. Laſtly, hang the ſtile directly ouer the ſubſtile, and parallel to it, at the diſtance formerly taken.

XXX Vſe. *How by* Sines and tangents *to calculate the* places of the Meridian, *and* Subſtile, *and the* height of the Stile *aboue it: and the* diſtance of the Meridian of the Æquinoctiall from the Subſtile; *together with the* places of houre lines, *both by calculation, and alſo Geometrically.*

I haue already taught the making of all manner of plaine Dyals moſt eaſily by the Inſtrument, for the ſame height of the Pole. But if any man either want an Inſtrument, or elſe deſireth greater exactneſſe, I will alſo here ſhew how to performe the ſame by calculation, on the other ſide of the Inſtrument.

In a plaine erect Dyall declining.

As the *Radius*
is to the Sine of *declination*;
So is the tang. of the compl. of the *Poles height*.
to the tang. of the *diſtance of the ſubſtile from the Meridian.*

Againe
As the *Radius*
is to the Sine of the compl. *of Declination*;
So is the Sine of the compl. of the Poles *height*,
to the Sine of the *height of the ſtile*
aboue the ſubſtile.

Thirdly
As the Sine of the *Poles height*
is to the *Radius*;
So is the tang. of *Declination*,
to the tang. of the *Diſtance of the Meridian of the Æquinoctiall from the ſubſtile.* And this diſtance is euer leſſe then 90 degrees.

In a plaine Eaſt and Weſt Inclining *and* Reclining Dyall.

As the *Radius*
is to the Sine of the compl. of { *inclination* / *reclination*
So is the tang. of the *height of the Pole*,
to the tang. of the *diſtance of the ſubſtile from the Meridian.*

Againe
As the *Radius*
is to the Sine of { *inclination* / *reclination*
So is the Sine of the *height of the Pole*,
to the Sine of the *height of the ſtile*
aboue the ſubſtile.

Thirdly,
As the Sine of the compl. of the *Poles height*,
is to the *Radius*;
So is the tang. of the compl. of { *inclination* / *reclination*
to the tang. of the *diſtance of the Meridian of the Æquinoctiall from the ſubſtile.* And this diſtance is euer greater then 90 degrees.

In

In plaine Dyalls *both* declining *and alſo* inclining, *and* reclining.

As the *Radius*
is to the Sine of { *inclination* / *reclination*
So is the tang: of *Declination*,
to the tang. of the compl. of *the diſtance of the Meridian, from the line parallel to the horizon.*

Againe
As the *Radius*
is to the Sine of the compl. of *Declination*;
So is the tang. of the compl. of the *Poles height*,
to the tang. of *Baſe I.*

If the Dyall be South recliniug, or North inclining, the *ſumme of Baſe I*, and of the complement of Inclination or reclination ſhall be *Baſe II.* But if the Plaine be South inclining, or North reclining, the *difference of Baſe I*, and of the complement of inclination, or reclination ſhall bee *Baſe II.*

Then ſay thirdly,
As the Sine of the compl. *of Baſe I*,
is to the Sine of the *compl. or exceſſe of Baſe II*;
So is the Sine of the *height of the Pole*,
to the Sine of the *height of the ſtile aboue the ſubſtile.*

Fourthly,
As the Sine of the compl. of *the height of the ſtile aboue the ſubſtile*,
is to the Sine of *Declination*;
So is the Sine of the compl. of the *height of the Pole*,
to the Sine of the compl. of the *diſtance of the ſubſtile from the line parallel to the horizon.*

Fiftly

As the Sine of the compl. of the *height of the ſtile aboue the ſubſtile*,
is to the Sine of *Declination*;
So is the Sine of the compl. of { *inclination* / *reclination* }
to the Sine of the *diſtance of the Meridian of the Æquinoctiall from the ſubſtile.*

And note that in South reclining, and North inclining Plaines, if *Baſe II* be leſſe then a quadrant, the contrary pole is eleuated aboue the Plaine: And if *Baſe II* be equall to a quadrant, the Plaine doth cut the Pole of the Æquinoctiall.

Now concerning the placing of the ſubſtile vpon the Dyall plaine (as I haue already in the *XXVIII Vſe* ſhewed for the Meridian) Wee are to know, Firſt that the ſubſtile is to be drawne vpward from the line parallel to the horizon, if the Plaine be Northerne; or downeward from it, if it bee Southerne. Except in North reclining, and South inclining Dyalls, in which the *Baſe I* exceedeth the complement of inclination, and reclination: for in them it is quite contrary. And ſecondly that the ſubſtile is to be drawne in the contrary ſide from the Declination. But in North inclining, and South reclining Dyalls, in which the contrary Pole is eleuated, the ſubſtile muſt be drawne through the center into the oppoſite quadrant of your Dyall circle.

Laſtly, the *houre lines* in all manner of plaine Dyalls, are thus to be found.

If the ſubſtile and houre bee both on the ſame ſide of the Meridian: the arch of the Æquinoctiall betweene the ſubſtile, and the houre line, ſhall bee equall to the difference of the two diſtances, namely of the houre line from noone, and of the Meridian of the Æquinoctiall from the ſubſtile. But if the ſubſtile bee vpon one ſide of the Meridian,

ridian, and the houre on the other ſide: it ſhall be equall to the ſumme thereof. Then ſay

As the *Radius*
is to the Sine of the *height of the ſtile aboue the ſubſtile*;
So is the tang. of the *arch of the æquinoctiall, betweene the ſubſtile and the houre line*,
to the tang. of the *arch of the circle of your Dyall plaine, betweene the ſubſtile and that lower line.*

Or elſe you may without calculation Geometrically inſcribe the houre lines in Dyals hauing centers (for how to doe it in Dyals not hauing centers, I haue already ſhewed in the *XIX Vſe*) thus.

Deſcribe in your Dyall plaine a line for the ſtile, at the ſame height or diſtance from the ſubſtile, that the true ſtile ought to haue. Take alſo in the ſubſtile (a reaſon you ſhall ſee) a point, and through it draw at right angles a long line, for the contingent, or Æquinoctiall interſection. Againe from the ſame point let fall a perpendicular vnto the ſtile: the length of this perpendicular is the neareſt diſtance betweene that point and the ſtile: and it is alſo the diſtance of the center of the Æquinoctiall from that point: meaſure it therefore vpon the ſubſtile, the contrary way from the center of the Dyall: and hauing thus the center of the Æquinoctiall, deſcribe thereupon toward the contingent line one halfe of the Æquinoctiall circle: which if the ſubſtile be the Meridian, or 12 a clock line of your Dyall, you muſt begin to diuide into houres at the ſubſtile: But if the ſubſtile and Meridian of your Dyall be ſeuerall lines, apply a ruler to the center of the Æquinoctial, and to the interſection of the 12 a clock line with the contingent, and there draw a line: this line ſhall be the Meridian of the Æquinoctial: at which you muſt begin to diuide the Æquinoctiall circle into houres, both wayes. Then applying a ruler vnto the center of the Æquinoctial & euery one of thoſe diuiſions, where the

the ruler in euery place ſhal cut the contingent line, there make a marke: and laſtly, through euery one of thoſe marks from the center of the Dyall, draw the houre lines themſelues.

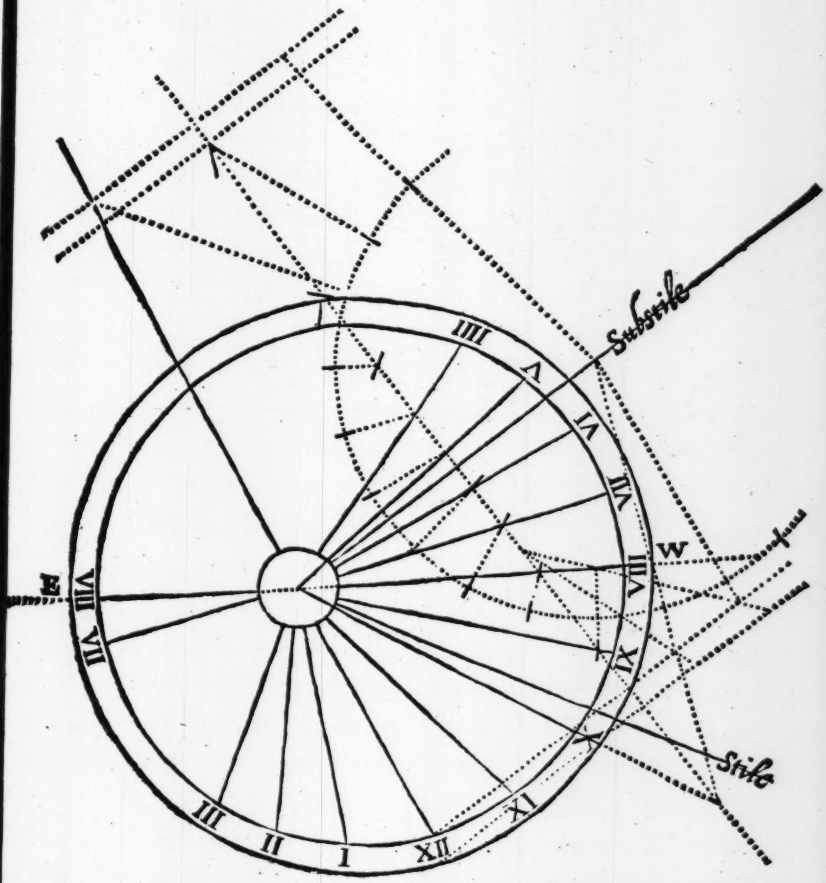
And if in any houre line it ſhall happen, that the ruler ſo applied will not reach to interſecat the contingent line: you may thus help your ſelfe. Which rule alſo may ſerue you to find the Meridian of the Æquinoctiall, as often as the interſection of the Meridian of the Dyall with the contingent, falleth without your paper or plaine.

Draw the houre line as farre as it will goe. And take with your Compaſſes the diſtance of the interſection point of the contingent with the ſubſtile, both from the center of the Dyal, & frō the center of the Æquinoctial. And taking at all aduenture a point in the contingent line, on that ſide in which the houre line is, meaſure from that point on the contingent, both thoſe diſtances: and at the ends of them both draw two lines parallel to the ſubſtile, croſſing the contingent. Then applying a ruler to the point, which you tooke at all aduenture, and to the interſection of the parallel, which hath the diſtance of that center, whence the houre line giuen proceedeth, with that houre line: where the ruler ſhall cut the other parallel, make a prick: and meaſure the diſtance betweene that prick and the contingent, vpon the former parallel, on the other ſide of the contingent. Laſtly, out of the proper center through the end of that meaſure, draw a line: which ſhall be that you deſire.

An example of this Geometricall way of delineating the houre lines you ſhall finde in the deſcription of a South vpright Dyall declining 35 degr. and reclining degr. 41 min. 30. by conſidering whereof theſe rules will be found exceeding plainely ſet downe: As alſo all the other rules and obſeruations here deliuered, to one that is any whit pregnant and ingenious, will neede no other exemplification, then the inſpection of the inſtrument it ſelfe, and of theſe ſeuerall Dyalls following

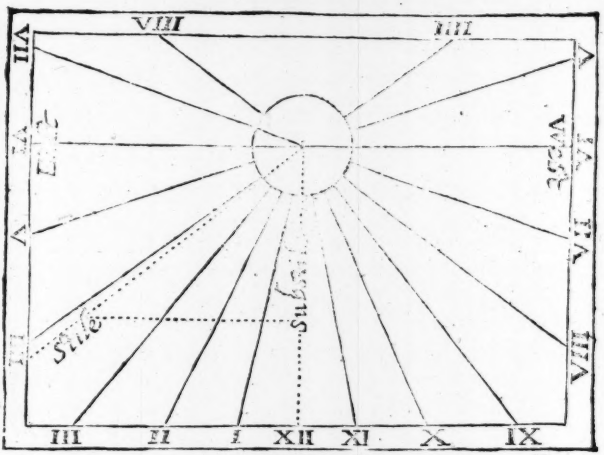
FINIS.

A North Dyall declining Eaſtwards 15 deg. reclining 41 deg. 30 min. Latitude 51 deg. 30 min.

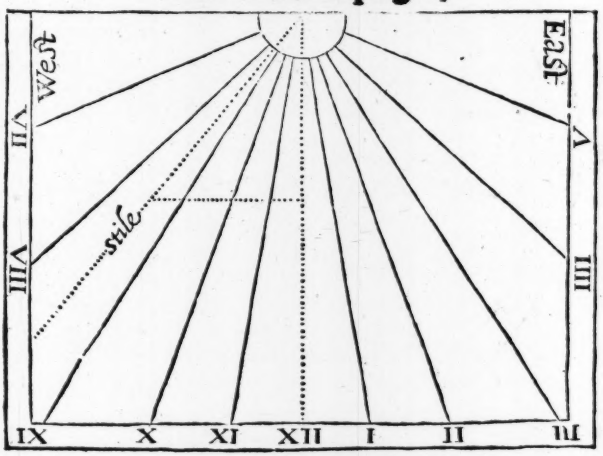


X A Mori-

A Horizontall Dyall.

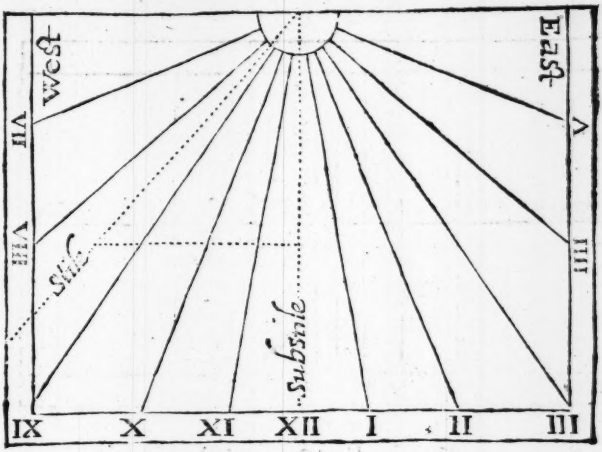


South direct vpright.

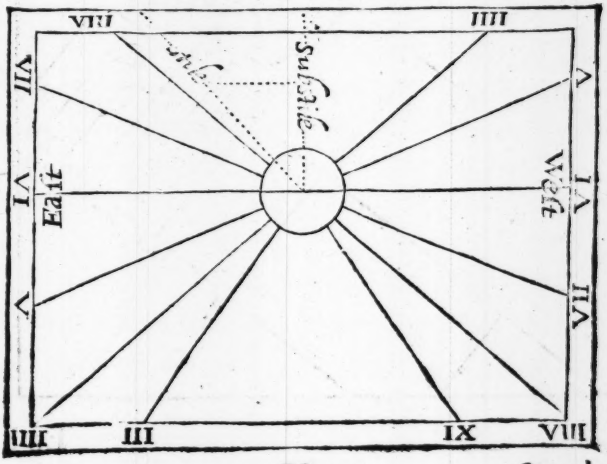


South

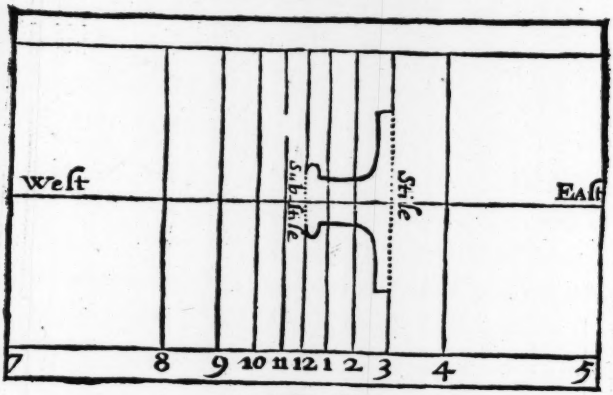
South direct inclining 24 deg.



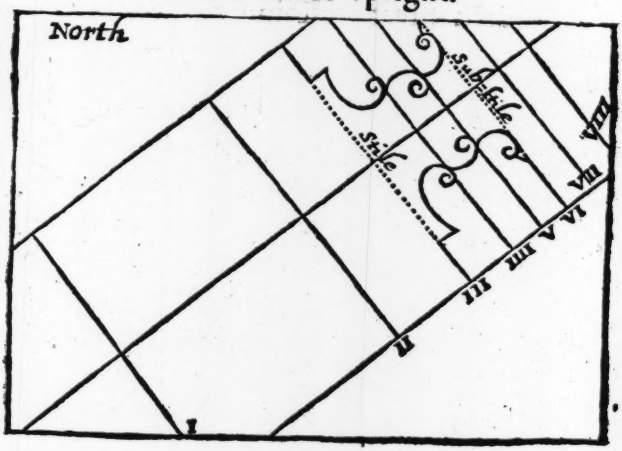
North direct reclining 24 deg.



South direct reclining æquall to the com-
plement of the poles height.

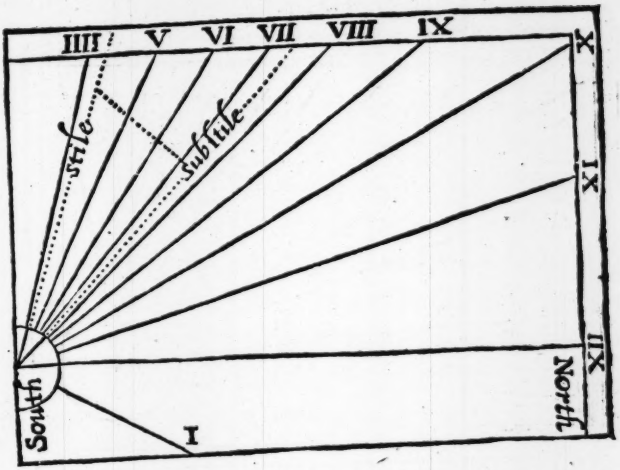


West direct vpright.

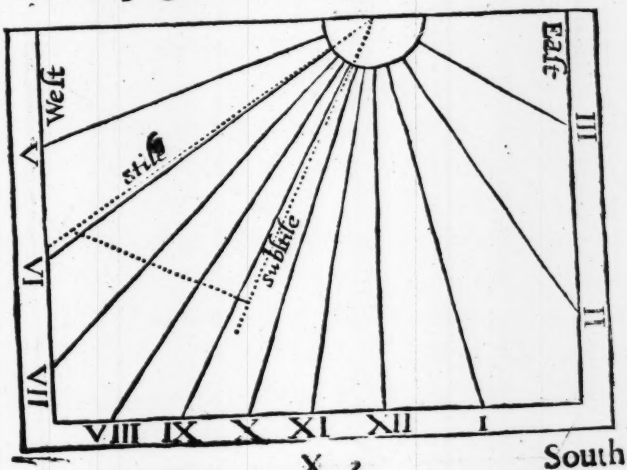


East

East direct reclining 3 2 deg.



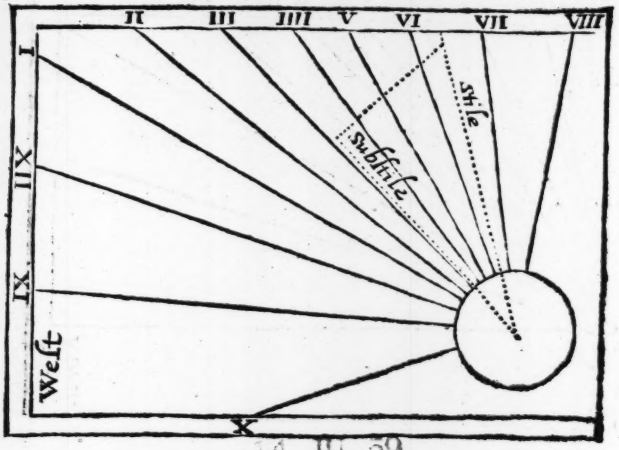
South vpright declining Eastward 3 5 deg.



South

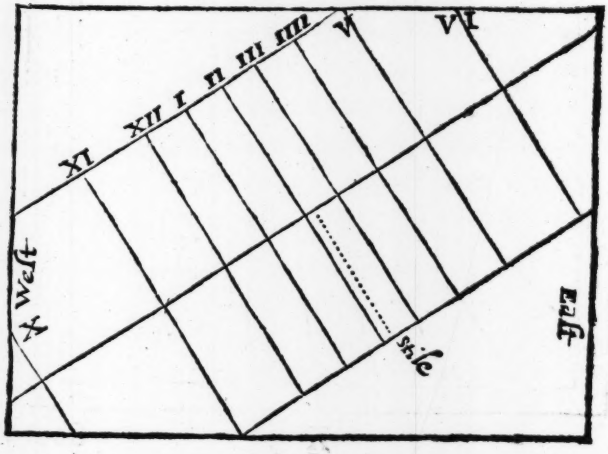
South declining Westward 76 degr.

reclining. 48 deg.



South declining Westward 61 degr.

Reclining 21 deg. 5' min.



The Translator to the Readers.

Courteous Readers, by reason of my absence whilest this worke was in the Presse, some faults haue escaped, which otherwise might happily haue bin avoyded: for the Compozer being vnacquainted with that manner of *Æquations expressed by letters*, where any *fraction* was on the one side, he misplaced them a little out of order, which ought to haue bin set thus

$$\frac{\text{R. mult}^{a} - 1 \text{ in R in } a}{D} = z$$

Other small faults, either letters mistaken, or points misplaced, I request you to amend, where you find them amisse: the chiefest that I haue yet noted are these which follow; and are thus to be corrected.

Pag. 11, in the table for, ☊ make ☋. *pag.* 16 *lin* 24 vnto 108 $\frac{33}{}$ +; *pag* 35 *lin.* 5. 61 $\frac{42}{144}$ *p* 36, *lin.* 11 $17\frac{48}{}$ *p* 36, *lin.* 14 $17\frac{48}{}$. $31\frac{36}{}$ + . *p.* 41, *lin.* 9, $1\frac{0472}{}$

pag. 53, *lin.* { to cylindrical / 18 it false, why / 20 to confirme an error?

pag. 56 *lin* 6, 4 roodlands *& lin* 14 diminutiue *pag* 57. *l.* 14, opinion is, that at London a Cylindrical vessell *pag.* 94, *lin.* 21, a circle, or 90 degrees.

pag 99, In the triangle *VI*, the small line in the angle *B*, shewing it to be giuen, is wanting. *pag* 100 After the triangle *VII line* 3, and then the side *DC*, by the *III*.

And in the triangle *VIII*, set *A* at the end of the pricked perpendicular line. *Pag* 142, *line* 10, to the tangent of the arch of *pag* 143, *line* 3, In North reclining, and South inclining planes *p.* 152, *line* 30, North Dyall declining Eastwards 35 degr.

AN ADDITION VNTO THE VSE OF THE INSTRVMENT CALLED THE CIRCLES OF PROPORTION, For the Working of *Nauticall Queſtions*.

Together with certaine neceſſary Conſiderations and Advertiſements touching NAVIGATION.

All which, as alſo the former Rules concerning this Inſtrument are to bee wrought not onely Inſtrumentally, but with the p nne, by Arithmeticke, and the Canon of *Triangles*.

Hereunto is alſo annexed the excellent Vſe of two Rulers for *Calculation*.

And is to follow after the 112 Page of the firſt Part.

LONDON,
Printed by AVGVSTINE MATHEWES.
1633.

OF NAVIGATION.

CHAP. I.

Certaine generall Advertiſements concerning the uſe of this Inſtrument; together with the deſcription of ſuch Circles as are newly added thereto, ſerving for Navigation.

The firſt part of this Chapter.

WHen I penned the rules which have been formerly ſet out to ſhew the uſe of this Inſtrument, I was carefull to doe it with as much plaineneſſe and perſpicuity, as might be in a ſubject not as yet obvious to vulgar knowledge, ſo that any one but moderately exerciſed in Arithmeticke and Geometrie, might (as I conceived) apprehend the workes and practices taught therein. But being ſince certifyed that ſome few difficulties ſeeme, or indeed rather are feared, to be in the manner of the delivery of theſe Rules: I thought it would not bee impertinent, and alien from this preſent purpoſe, if in the very beginning I ſhall endeavour to explane ſuch doubts, for the ſatisfaction of any that ſhall ſticke thereat.

The ſcruples, which chiefely ſeeme to cauſe their "difficultie, are theſe two: Firſt, that the parts or fra-

"ctions are not ſet downe with their Numerator and "Denominator, as is uſually done; but are conteined "with the whole Numbers, as it were in one ſumme, "with a ſmall rectangular line only between them to ſe- "parate the parts from the Integers. And ſecondly, moſt "of the examples are not wrought at large, but the ſum- "marie and finall reſolution thereof briefely intimated. The former of which two ſcruples ariſeth from the ignorance of the true nature and manner of Decimall fractions: and the latter, from want of rightly conſidering the Rules, whereby the valure of the number emergent or found out by proportion, and other Arithmeticall operations, is eſtimated: which are thoſe that are delivered in the ſecond and fifth Chapters of the firſt part of that booke.

That wee may the better conceive the nature of *Decimall fractions*, let us imagine a line either ſtraight or circular, of any length, bee it a foot, or a yard, or one degree, or many; or elſe an houre, or a day, or any other continuity. This being conſidered in it ſelfe intire and undivided is an Vnite or one whole thing of that kind, as one foot, one degree, one houre, &c. Then imagine that Vnite or whole to bee divided to 10 equall parts, that whole ſhallbee 10. Againe imagine every one of thoſe tenth partes to bee ſubdivided into 10: the whole ſhall bee 100, and each firſt diviſion ſhall bee 10: and theſe ſecond diviſions ſhall bee hundreth partes. Thirdly, imagine every one of thoſe hundreth partes ſubdivided into 10: the whole ſhall bee 1000, and each firſt diviſion ſhall bee 100: and each ſecond diviſion ſhall bee 10: and theſe third diviſions ſhall bee thouſanth parts. And ſo proceeding in this Decimall ſubdiviſion, you may in your imagination divide the Vnite or whole into ten-thouſanth parts, and hundred-thouſanth parts, and

and millioneth parts, and ſo infinitely. And ſo that ſegment which in the firſt diviſion was 10, 20, 30, &c. ſhall in the ſecond diviſion bee 100, 200, 300, &c. and in the third diviſion 1000, 2000, 3000, &c. As for example,

$\frac{3}{30}$ is $\frac{30}{100}$ or $\frac{300}{1000}$ or $\frac{3000}{10000}$: and $\frac{45}{100}$ is $\frac{450}{1000}$ or $\frac{4500}{10000}$: and $\frac{374}{1000}$ is $\frac{3740}{10000}$ or $\frac{37400}{100000}$:&c.

Hence followeth that you may encreaſe the Numerator of any Decimall fraction by putting thereto as many cyphers or circles as you pleaſe without altering the quantity thereof, ſo that alſo you joyne ſo many cyphers to the Denominator.

Now therfore *a Decimall fraction is that which hath for his Denominator the figure* 1 *with one or moe circles after it*, as 10, 100, 1000, &c. And ſeeing the uſe of the Denominator in a fraction is to ſhew into how many ſuch parts the whole or Vnite is divided: if otherwiſe by any convenient ſigne the Denominator may eaſily and certainely bee knowne by the Numerator onely, it will bee a needeleſſe labour ſtill to ſet it downe.

The moſt fit and convenient ſigne to know the Denominator of a Decimall fraction is by *a ſeparating Line*. For if the number mixed of integers and parts be written togeather in one ranke, with a ſmall rectangular line drawne next after the Vnite place, cutting off the parts from the Integers: the number of figures or places in the parts ſo cut off ſhall ſhew how many circles or cyphers are to bee ſet after 1 in the Denomina-

tor As for example, $3700\frac{6}{\circ}$ is all one with $3700\frac{6}{10}$, that is 3700 Vnits, and sixe tenth parts. Againe $370\frac{06}{\circ}$ is all one with $370\frac{6}{100}$, becauſe after the ſeparating line follow two figures 06. Likewiſe $37\frac{006}{\circ}$ is all one with $37\frac{6}{1000}$, becauſe three figures 006 follow the ſeparating line. Alſo $3\frac{17006}{\circ}$ is all one with $3\frac{17006}{100000}$. And $0\frac{37006}{\circ}$ is all one with $\frac{37006}{100000}$, that is no unite at all, but that fraction only. And $0\frac{037006}{\circ}$ is all one with $\frac{37006}{1000000}$, becauſe after the ſeparating line are ſixe places of figurss 037006. By all which diverſities of placing the ſeparating line it is apparant that *the number of circles in the Denominator of any Decimall fraction muſt bee equall to the number of places of figures following the ſeparating line.*

Wherfore though there be no Vnite, but that it be a pure fraction, yet *it will be convenient to note the Vnite place with a circle before the ſeparating Line*; that ſo the value of the fraction, through the number of places therein may more plainely appeare.

And beſides that the ſetting of Decimall parts thus in one line with the Integers, hath more concinnity and neatneſſe with it, then either with a Denominator, or by noting (as ſome have done) with ſmall figures the primes, ſeconds, thirds, and the reſt. Theſe fractions both mixt and pure are ready without any further reduction, for any Arithmeticall operation.

For in *Addition* and *Subduction*, the numbers given, being fitted together by their ſeparating lines, having the like places or degrees ſet under one another, each in their owne file, may be added or ſubducted in the very ſame manner as if they were all whole numbers.

And in *Multiplication* the numbers given being multiplyed

tiplyed one by the other, according to the usuall manner of whole numbers, the product found out shall have so many places of parts, as are in both the numbers multiplyed.

And in *Division* the ordinary manner of whole numbers is to bee used; onely remembring that every figure of the Quotient shall be of that degree, whereof that figure of the Dividend is, under which the Vnite place standeth in the finding out of it, is.

Thus have I with as much plainenesse and brevitie as possibly I could cleered *the first scruple, by shewing the true reason of Decimall fractions.*

The second conceived difficultie is for not setting downe at large the operation of most of the Examples, but onely of some few here and there.

It is true that in every worke I doe not say (as some have done) bring that hither, or remove this thither: But having first taught the manner of working proportions upon the Instrument, and also delivered proper rules for particular questions, and wrought at the full summe of the hardest, I would not in every Example shew the like punctuallnesse, that neither I might blunt the edge and industrie of the ingenious Practicer with too much easinesse, nor the Booke grow to an enormous bulke and greatnesse.

That therefore the studious Reader may not need such verbosity and tædious instructions, he is to be advised oftentimes (and that attentively) to peruse the first chapter of the first part, where the *description and use of the severall circles are declared:* and also the second Chapter concerning *the working of proportions,*

and

and of *Multiplication* and *Diviſion*: and therein thoſe *foure Conſiderations*, or *Rules for finding out the true value of the fourth or emergent number ſought for*: And thereto the fifth Chapter of the *quadrating* and *Cubing of numbers*. For in aſſigning a true quantitie unto the Emergent number lyeth the greateſt difficultie of this operation, eſpecially if the worke bee in the fourth Circle.

In *Signes* and *Tangents* it is not altogether ſo hard, becauſe all the revolutions or circuits of both are actually ſet downe in ſeverall Circles.

The *Signes* have two Circles, which in this *new additament for Navigation* are theſe; The *tenth Circle* from about 35 minutes, unto 6 Degrees; and the *Firſt* from 6 Degrees, to 90, the end of the Quadrant.

The *Tangents* have foure Circles: namely the *Ninth* from about 35 minutes to 6 Degrees. The *Second* from 6 Degrees to 45. The *Third* from 45 Degrees to 84. And the *Eighth* from 84 Degrees till about 89 Degr. and 25 minutes.

But the *fourth Circle* being actually but one, doth potentially containe all Degrees and places both of Integers and Decimall parts. For the nine figures written in the ſpaces may ſignify unites, or tennes, or hundreds, &c. or elſe tenth parts, or hundreth parts, or thouſanth parts, &c.

If any *number* be to be conſtituted upon the *fourth Circle* of the Inſtrument, take evermore one of thoſe nine figures in the ſpaces for the firſt ſignificant figure of that number: and among the ſubdiviſions thereof reckon the true poynt or place of the number propoſed.

As

As if 2 were propoſed: ſeeke the figure 2 in the ſpaces, & upon that lme ſet one arme of the Index. Again if 375 be propoſed: ſeeke the firſt figure 3 in the ſpaces: and in the ſubdiviſions from 3 towards 4 account 75: and at the end thereof ſet one arme of the Index. Likewiſe if 01092 bee propoſed: becauſe the two Circles are not ſignificant, ſeeke the figure 9 in the ſpaces: and in the greater diviſions thereof from 9 towards 1 account 2, and there ſet one arme of the Index.

If any *ratio* bee propoſed to bee taken on the Inſtrument: ſet the two armes of the Index upon the two termes of the ratio found out, as was even now taught. Then conſider the diſtance or arch betweene thoſe two termes, counting from the place of the Antecedent to the place of the conſequent forward, or according to the order of the figures, if the antecedent terme bee leſſe then the conſeqnent: Or elſe backward, contrary to the order of the figures, if the antecedent bee greater.

This diſtance or arch betweene the places of the two termes in the Inſtrument (which is alſo the aperture of the armes of the Index) I may fitly call the *Inſtrumentall difference*: but it is not evermore the *reall or true difference*: which alſo is moſt needefull to bee knowne. The rules whereof are theſe three.

Firſt, if either the numbers given be of the ſame degree: Or if they differ but one degree, and the *line of the Radius* fall betweene the places of the two termes in the Inſtrument: the *Inſtrumentall difference* ſhall alſo be the *true and reall*.

Secondly, if the numbers given bee not of the ſame degree, and the *line of the Radius* fall not betweene

the places of the two termes in the Inſtrument: looke how many degrees the numbers differ one from the other, ſo many whole circuits of the *fourth Circle* ſhall bee added to the *Inſtrumentall difference* to make the *reall or true difference*.

Thirdly, if the numbers given be not of the ſame degree, and the *line of the Radius* doth fall betweene the places of the two termes in the Inſtrument: looke how many degrees the numbers differ, ſo many whole circuits, wanting one, of the *fourth Circle* ſhall be added to the *Inſtrumentall difference to make the reall.*

As in example: If the *ratio* of 375 to 2 be propoſed: the ſame being taken upon the Inſtrument; the *true difference* betweene them, over and above the arch or angle of aperture, ſhall bee two whole circuits, by the ſecond rule. And if the *ratio* of 375 to 0,92 be propoſed: the ſame being taken upon the Inſtrument; the *true difference* betweene them, over and above the arch or angle of aperture, ſhall, by the third rule bee but three whole circuits, (although the termes differ foure degrees) becauſe the *line of the Radius* falleth within that arch, reckoning it from the antecedent arme to the conſequent backward.

Againe, the antecedent terme of any *ratio* being given, together with *the reall or true difference* (that is both the due aperture of the Index, and alſo the number of circuits) betweene the termes, and whether of the two bee the greater: it is alſo needefull to know how to eſtimate the conſequent terme. The rules whereof are theſe two.

Fourthly, if the true difference bee leſſe then one circuit, and the *line of the Radius* fall not betweene the

places

places of the two termes; the numbers are both of the ſame degree. But if the *line of the Radius* fall between them they differ one degree.

Fiftly, if the true difference containe one or more circuits, and the *line of the Radius* fall not betweene the places of the two termes; the numbers differ ſo many degrees as there are whole circuits. But if the *line of the Radius* fall betweene them they differ one degree more then there are whole circuits.

As in example: If the *ratio* of 375 to 2 be propoſed: and alſo another antecedent $0\frac{1}{092}$: unto which a proportionall conſequent is required to be ſought. Becauſe the *true difference* of $0\frac{1}{092}$ unto his conſequent in the Inſtrument is equall to the *true difference* of 375 to 2, that is two whole circuits more then the aperture: and the antecedent $0\frac{1}{092}$ is greater then the conſequent ſought for: ſet the antecedent arme of the Index upon $0\frac{1}{092}$, and the conſequent arme reckoning backeward, at the ſame aperture, will cut 494. But of what vallue or degree this fourth number is, is yet uncertaine. Now foraſmuch as the *reall difference* betweene the termes of the *ratio* propoſed is two whole circles above the aperture, as was ſhewed in the former example after the third rule; And in this preſent poſition of the Index the *line of the Radius* falleth not between the armes: the *difference of degrees* ſhall alſo bee two, by the fifth rule. Wherefore the firſt figure of 494 ſhall bee two whole degrees backeward from the firſt ſignificant figure of $0\frac{1}{092}$ that is $0\frac{1}{00049}$+ (*viz*) ſomewhat better then 49 hundred thouſand parts.

Againe, if the *ratio* of 375 to $0\frac{1}{092}$ be propoſed: and alſo another antecedent 2, unto which a proportionall conſequent is required to bee ſought. Becauſe the *true*

difference of 2 unto his conſequent in the Inſtrument, is equall to the *true difference* of 375 to 0|09? : and the antecedent 2 is greater then the conſequent ſought for. Set the antecedent arme of the Index upon 2, and the conſequent arme reckoning backward at the ſame aperture will cut 49+ as before. Now foraſmuch as the *reall difference* betweene the termes propoſed is three whole circuits above the aperture of the Index, as was ſhewed in the latter example after the third rule. And in this preſent poſition of the Index the *line of the Radius* falleth betweene the armes the difference of degrees ſhall be one more then three, that is foure *by the fifth rule*; wherefore the firſt figure of 49+ ſhall bee foure whole degrees backward from 2, that is 0|09|49+. I will conclude this part, with a ſummary recapitulation of all the former rules into theſe two branches.

The termes of a ratio being propoſed, to find the reall or true difference betweene their places in the fourth circle of the inſtrument. I. *If either the numbers given be of the ſame degree: or elſe differ but one degree, the line of* 1 *falling betweene them: they differ leſſe then a circuite.* II. *If the numbers bee not of the ſame degree: they differ ſo many whole ciruits as they doe degrees. But yet if the line of* 1 *fall betweene them: they differ one circuit leſſe.*

The antecedent terme of a ratio being given, together with the reall or true difference of the termes in the Inſtrument: to find out the conſequent terme. I. *If the reall difference be leſſe then one circuit, and the line of* 1 *fall not betweene the places of the two termes: the numbers are both of the ſame degree. But if the line of* 1 *fall betweene the places: they differ one degree.* II. *If the reall difference containe one or moe circuits: the numbers differ ſo many degrees as there are whole circuits. But if*

the

the line of 1 *fall betweene the places : they differ one degree more.*

Thus have I with as much perſpicuouſneſſe as I am able, explained the generall rules of working by this Inſtrument, which have beene delivered in the *firſt*, *ſecond*, and *fifth* Chapters of the *firſt part :* and exemplifyed the documents with as hard exampies as any I could bethinke my ſelfe of. And now I ſuppoſe the ſolertious practizer will bee able eaſily to finde out a fourth proportionall unto any three numbers given, and certainely to eſtimate the value thereof: ſo that now he will not be troubled for want of working the Queſtions at large.

For the uſe of Navigation are added two circles, the ſixth and the ſeventh : and a ſmall alteration in the fifth. For the *fifth circle* is here divided alſo into 50 parts: and is conceived to have *two circuits*. The *firſt circuit* is unto 50 : The *ſecond circuit* from 50 unto 100. Wherefore the *figures are doubly noted :* on the *neerer ſide* of the long lines of tenth diviſions are ſet 10, 20, 30, 40, 50, for the *firſt circuit :* And on the *further ſide* of thoſe lines are ſet 60, 70, 80, 90, for the *ſecond circuit*. And the ten ſubdiviſions in every one of thoſe 50 parts are the *Decimall parts* thereof. *The ſecond part of this Chapter.*

The *ſixth* and *ſeventh circles* are divided into degrees: and every degree into ten parts, containing 6 minutes, or rather 10 hundreth parts a piece, The *ſixth circle* hath the degrees unto 44½ : and the *ſeventh circle* hath from 44½ unto 70. And theſe degrees ſerve for ſo many ſeverall *Latitudes*, or *Elevations of the Pole*.

The manner of uſing theſe circles is double. Firſt, *Two Latitudes being given in the ſame Hemiſphere, that*

is both Northerne, or both Southerne, to find the ſumme of all the Secants betweene them. Set one arme of the Index upon one *Latitude* and the other arme upon the *other*; then remove the arme that ſtood upon the *leſſer Latitude* unto the *line of the Radius*: and the other arme with the ſame opening, ſhall in the *fifth circle* give the *number of Secants betweene the two Latitudes propoſed.* As if the *number of Secants* betweene theſe two heights of the Pole $48\frac{1}{3}$ and $56\frac{1}{7}$ bee deſired. Set one arme of the Index upon $48\frac{1}{3}$ and the other arme upon $56\frac{1}{7}$: then remove that arme that ſtoood upon $48\frac{1}{3}$ unto the *line of the Radius*: and the other arme with the ſame opening, ſhall in the *fifth circle* give $131\frac{85}{3}$, the *number of Secants betweene the two Latitudes propoſed.*

Secondly, *The ſumme of all the Secants between two Latitudes in the ſame Hemiſphære being given, together with one of the Latitudes, to find the other Latitude.* Set one arme of the Index on the *line of the Radius*, and open the other arme unto the *ſumme of Secants given (in the fifth circle)*: then remove the arme that ſtood on the *line of the Radius* to the *Latitude* given, if it be the *leſſer*: or if the *Latitude* given be the *greater*, remove that arme that ſtood at the end of the *ſumme of the Secants*, unto that *greater Latitude*: and the other arme at the ſame opening ſhall give the other *Latitude.* As if there be given $131\frac{85}{3}$ the *ſumme of Secants* from the *Latitude* of $48\frac{1}{3}$ to the Pole-ward: Set one of the armes of the Index on the *line of the Radius*, and the other arme at $131\frac{85}{3}$ in the *fifth circle.* Then remove the arme that ſtood at the *line of the Radius*, unto the *Latitude* $48\frac{1}{3}$: and the other arme, at the ſame opening ſhall point to $56\frac{1}{7}$ *the degrees of the other Latitude ſought for.* Againe, if the ſame *ſumme of Secants* $131\frac{85}{3}$, with the *greater Latitude* $56\frac{1}{7}$ degrees, be given: ſet one of the armes of the Index on the *line of the Radius*,

and

and the other arme at 13|853 in the *fifth circle* Then remove the arme that ſtood at 13|853, unto 56|7 degr: the *greater Latitude*, and the other arme, at the ſame opening ſhall cut 48|3 deg: which is the *leſſer Latitude* ſought for.

And *if the two Latitudes be in the ſeverall Hemiſphæres, that is one Northerne and the other Southerne, the manner of working differeth in effect but little from the former.* As if the *ſumme of the Secants* betweene theſe two heighrs of the Pole, *viz.* 6|5 on the *North ſide of the Æquinoctiall*, and 13|4 on the *South ſide* bee deſired. Set one arme of the Index *on the line of the Radius*, and the other arme on *either of the Latitudes given*, ſuppoſe on 6|5. Then bring that arme on 6|5 unto *the line of the Radius*: and where the other arme, at that opening chanceth to light, there hold it faſt: and open the arme that ſtandeth on *the line of the Radius*, unto the *other Latitude* 13|4. Afterward bring the arme that ſtood on the *former Latitude* 6|5 unto the *line of the Radius*, and the other arme, at the ſame opening, ſhall in the *fifth circle* cut 20|°37, the *ſumme of the ſecants ſought for.*

Laſtly, *the ſumme of all the Secants betweene two Latitudes, of which one is on the North ſide of the Æquinoctiall, and the other on the South ſide, being given; together with one of the Latitudes, to find the other Latitude:* As if the *ſumme of the Secants* bee 20|°37 and the *Latitude degr:* 6|5. Set one of the armes of the Index at the *line of the Radius*: & open the other arme unto 20|°37 in the *fifth circle*: and keeping the ſame aperture, bring this latter arme unto the *Latit:* 6|5: and where the former arme ſhall light, there hold it faſt, drawing in the latter arme to *the line of the Radius*. Laſtly with this new opening bring the other arme to *the line of the radius*: and ſo ſhall you find 13|4, the other *Latitude ſought for.*

Or

Or elſe peradventure *you may more eaſily find out the ſumme of the Secants betweene any two Latitudes given*, thus: Set the edge of the Index upon one of the *Latitudes*: and looke what diviſion it cutteth in the *fifth circle*: *keepe it* in mind. Againe, ſet the edge of the Index upon the other of the *Latitudes*: and looke what diviſion it cutteth in the *fifth circle*: *keepe that* in mind alſo. Theſe *two numbers kept in mind* are *the ſummes of the Secants for the two Latitudes given*: And are to bee *ſubducted one out of the other*, if the *Latitudes are both in the ſame Hemiſphære*: or elſe to be *added together*, *if the Latitudes are in diverſe Hemiſphæres*.

Alſo in like manner, *The ſumme of the Secants and one of the Latitudes being given, you may find out the other Latitude*, thus: Set the edge of the Index upon the *Latitude given*; and looke what diviſion it cutteth in the *fifth circle*. To this number *adde the ſumme of the Secants*, if the *leſſer of the two Latitudes* be given: Or elſe *out of* it *ſubduct the ſumme of the Secants*, if the *greater of the two Latitudes* bee given. But if the *two Latitudes are in the contrary Hemiſpheres*, the number found in the *fifth circle* is to be *ſubducted* out of the *ſumme of the Secants*. And ſo ſhall you have the *other Latitude*.

CHAP.

CHAP. II.

Of the Latitude, and Longitude of places in generall: and of keeping the account of time at Sea.

THE care and skill of the perfect Sea-man is to guide the ſhip at ſea unto any port that ſhal be deſired: which cannot be done unleſſe he bee able to find out in *in what place the ſhip is at any time.*

The *place of the ſhip at ſea is eſtimated and underſtood by comparing it with any knowne place: that is how much the ſame is ſituated from the place, where the Ship is, either toward the North or South, which is called the difference of Latitude: or elſe toward the Eaſt or Weſt, which is called the difference of Longitude.* For it being once knowne *how farre* any place upon the Globe of the earth is *wide of the Æquinoctiall* unto either Pole: and alſo *how farre the Meridian* of the ſame *is diſtant from the Meridian* of any knowne place: the *true ſituation* thereof is ſaid to be had.

The *Latitude* or diſtance of the place wherein the Ship is from the Æquinoctiall (which is all one with the height of the Pole there) *is taken by obſervation of the Meridionall altitude*, either of the *Sunne* by day, or of any *Starre* by night: as is not unknowne to almoſt every common Mariner: Or alſo by the 47 propoſition of the 20 Chapter of the firſt Part. And therefore being ſo vulgarly knowne, and taught of moſt that write of Navigation, I ſhall not need to ſpend time about it:

Especially my intent here being *to teach the use of my Instrument only, in tracing the Ships course.*

The *Longitude* of the place wherein the Ship is, that is the Easterly or Westerly distance of the Ship from the place whence the Voyage began, is the difficultie, and *Master-piece of Nauticall science*: Which hath set on worke the wits and inventions of many men, proceeding therein on diverse grounds.

For some have laboured to find the reason thereof by the *variation of the Magneticall needle*, supposing certaine Poles or points, unto which the ends of the needle doth in all places exactly respect. But besides that the *Meridian is difficultly to be had* with sufficient precisenesse, especially *at Sea, where the chiefest use of Longitude is*: the conceipt is only imaginary, without the warrant of any naturall principle.

Some considering the swiftnesse of the *motion of the Moone*, which is every day above 13 degrees, have supposed that either by the *true place of the Moone, to be observed* by exact Instruments; or else by the *moment of the Moones comming into the Meridian*, the *Longitude* might bee obtained. But neither the *true motion of the Moone is so exactly knowne, nor observation can at Sea bee so precisely made*, that any certaine truth in so *subtile a businesse* may be argued thereby.

Some have thought to observe the *Longitude* with *automata* or artificiall motions of long continuance: but not without great errour and hallucination.

Some by *Sand-glasses*, or *Waterglasses*: but both obnoxious to the diverse alterations and temperatures of the aire and climate wherein they are, especially that of

sand

ſand. The other by *water is more probable:* wherein I ſhould, in my judgement, preferre ſome *chymicall ſpirit* or liquor: becauſe it is not ſo ſubject to the impreſſion of the *aire.* And that there ſhould be *three glaſſes* uſed, one to runne, and two to receive ſucceſſively: That which runneth to be open above, to poure in the liquor, and to let in the aire, that the iſſue of the water be not hindred for want of aire to ſupply the vacuitie: The receivers to be cylindricall, with markes ſet on the outſide diſtinguiſhing houres and parts: and that there bee two of them, that when the liquor is come to the juſt height, another may inſtantly bee ſubſtituted, without loſſe of any liquor or time. This manner of obſerving the time is, in my opinion, the moſt likely of any that I know in uſe to conduce to the attaining of the *difference of Longitudes* of places. For by this meanes the *true time* in the place where the account beganne being knowne; and the *time by obſervation* of the Sunne or ſome Starre in the place, whither the Ship is come, being found; the *difference of thoſe times* reſolved into degrees of the Æquinoctiall will ſhew the *difference of Longitude* betweene the place of beginning the account, and the place where the Ship is, *Eaſtward*, if the exceſſe be of the time in the former place: or *Weſtward*, if the exceſſe be of the time in the preſent place of the Ship.

And in this manner of *keeping the reckoning of Longitude* it will bee expedient to make as *frequent obſervations* as the ſerenity of the skye will permit: that thereby your account may the rather bee freed from ſuch ſubreptious errours, which elſe will bee very incident.

This or any ſuch way of keeping the time, which ſhall by experience bee found moſt certaine (untill it ſhall pleaſe God to open a more naturall and proper

way for the discovery of *Longitude*) I would advise were carefully, and with a kind as it were of religious diligence practised in all, specially long voyages: and that in computing and tracing the *course of the Ship* by the *Compasse* and *log-line*, it also together with the *Latitude observed* be discreetly called into consultation.

Chap. III.

Of the Mariners Compasse, *and* Rumbes *or points thereof: and of* finding the circuit of the earth in miles.

THere be *foure things therfore whereof a Seaman should be most carefull & circumspect*, that he may happily with prosperous successe and a good conscience performe his intended voyage: First *the angle of inclination with the Meridian, on which the Ship maketh her course: which angle is directed by the Compasse:* and is commonly called the *Rumbe or poynt of the Compasse*. For the ordinary Mariners (by a rude and grosse division of the *Horizon* into 32 parts) observe 32 points, whereof foure are cardinall; other foure halfe points; eight are quarter points; and sixteene are by points. Others more curiously divide each point into foure parts making in all 128, which they denominate by a quarter, an halfe, and three quarters of a point. *A point containeth degr:* $11\frac{1}{4}$, that is degr: 11, min: 15, or degr: $11\frac{25}{100}$: and *a quarter of a point therfore is degr:* $2\frac{13}{16}$, that is degr: 2, min: 48$\frac{75}{100}$, or degr: 2$\frac{81}{100}$. By the continuall addition of which number this *table of Rumbes* ensuing is composed.

THE

Rumbs.	Rumbs.	Grad.	Gr. min.	Rumbs.	Rumbs.	
NORTH.		The Meridian Line.		SOVTH.		
		2 8125	2 48 75			
		5 1625	5 37 15			
		8 1437 5	8 26 25			
N b E	N b W	11 125	11 15	S b W	S b E	1
		14 0625	14 3 75			
		16 1875	16 52 15			
		19 0687 5	19 41 25			
N N E	N N W	22 15	22 30	S S W	S S E	2
		25 1312 5	25 18 75			
		28 1125	28 7 15			
		30 1937 5	30 56 25			
N E b N	N W b N	33 75	33 45	S W b S	S E b S	3
		36 0562 5	36 33 75			
		39 1375	39 22 15			
		42 1187 5	42 11 25			
N E	N W	45	45 00	S W	S E	4
		47 8312 5	47 48 75			
		50 0625	50 37 15			
		53 1437 5	53 26 25			
N E b E	N W b W	56 125	56 15	S W b W	S E b E	5
		59 0062 5	59 3 75			
		61 1875	61 52 15			
		64 0687 5	64 41 25			
E N E	W N W	67 15	67 30	W S W	E S E	6
		70 1312 5	70 18 75			
		73 1125	73 7 15			
		75 1937 5	75 56 25			
E b N	W b N	78 175	78 45	W b S	E b S	7
		81 0562 5	81 33 75			
		84 1375	84 22 15			
		87 1187 5	87 11 25			
East	West	90	90 00	West	East	8

The ſecond is the *meaſure of the Ships way on the Rumbe or point*, which is ordinarily reckoned in *miles*; ſuppoſing *a mile* on earth to anſwer to *a minute of a degree*; and that 60 *miles on a great circle give the difference of one whole degree*. But I rather reckon the *way of the Ship in hundreth parts of a degree*, and have framed my rules of Navigation thereto: becauſe this hath a more eaſy and convenient calculation then that by ſexageſme parts: and as I beleeve (for ſo I would have it) will hereafter grow into publike uſe. This *meaſure or quantity of the Ships way is found by the Logg-line and minute-glaſſe*.

The other two are, The *obſervation of Latitude as oft as it may be* for the weather: and the *keeping of time*: Of both which I ſpake ſufficient for my purpoſe in the former chapter. The two former, that is the *Rumbe and way of the Ship*, more properly fall within my preſent conſideration. For theſe are the *continuall companions and faithfull guides of the Sea-man*, which muſt direct him ſtill in ſhaping his courſe: unto theſe therefore hee muſt applie his ſtudie, and acquaint himſelfe moſt familiarly with them.

And firſt for his *compaſſe* he muſt be carefull or rather ſcrupulous that it be *exactly made*, and not bungled up, as thoſe uſually are, which are made for ſale: but that they be framed by ſome skillfull and conſcionable Artificer. The manifold cautions which are fit to bee had therein, are very gravely advertiſed by that *reverend Divine and learned Mathematician Maſter William Barlow in his Navigators ſupply* neere the beginning.

And as he is in the *making* of his *compaſſe* to ſhew his *arte*, ſo ſpecially in the *uſing* thereof he muſt exercife all *induſtrie* and *diligence*, that the *courſe be ſteered aright*, and

and kept to the *just point or Rumbe:* and not to commit his owne and all his companies safety, and the good successe of the voyage to the *negligence of a loose and idle Steeresman:* whereby it cannot be but that the account of the Ship shall be much confounded, and made uncertaine.

Againe for *measuring of the quantity of the ships way, It must first be knowne how many English feet of 12 inches to the foot, answer to one degree of a great circle upon the earth.* For if this be enormously mistaken, it cannot bee that the computation of the Ships course shall agree with the observations: but must needes make a maine difference, to the amazement of the Sea-man, and the casting of the whole Ship and company into unforeseene dangers.

Now an *English mile by statute is the length of* 8 *furlongs:* and every *furlong is* 40 *perches:* and *a perch is feet* $16\frac{1}{2}$: so that by this reckoning *a mile containeth* 5280 *feet in length:* though it be *usually* taken, or rather *mistaken, that* 60 *of such miles make a degree* (which would bee very strange, that our *English mile drawne from Barley cornes* should so happily fall out to answer to one minute) yet the truth is that *above* 66 *of our miles answer to a degree,* as by the observations of the most diligent enquirers is found out: so that in voyding of every ten degrees above one degree is lost: which is a maine enormity. But of this enquirie it will not bee amisse from our purpose if we shall a little discourse.

Diverse wayes by diverse Artists have beene practised *for finding out the true compasse of the earth:* And I know not whether any have given full satisfaction therein: but either the *grounds* they have *wrought on* have beene *uncertaine;* or the *distances* of the places *of observation too short;* or the *dilligence of the practiser* to bee suspected. That *way which is by the height of an hill, and a tangent*

line

line from thence to the ſuperficies of the ſea, is rather a phantaſie, then a thing of actuall performance. For neither the perpendicular height of the hill above the levell of the water can with any certainty bee obtained: nor ſuch a tangent line by reaſon of the *refraction of the vapours* continually riſing out of the ſea can be eſtimated.

But it would for the performance hereof *be an excellent worke, if the height of the Pole at two townes of this Land, diſtant North-ward one from the other ſome ſcores, or rather hundreds of miles, being with Inſtruments of ſufficient magnitude by ſome learned Artiſts exactly obſerved: there were alſo imployed certaine skilfull Surveyors (ſuch as are indeed lovers of art and truth) to take the true diſtances and poſitions from place to place betweene the ſaid townes.* Which *ſurvey* I could wiſh were made *with good plaine tables*, and with the ſame ſcale, which ſhould not be leſſe then a foot by ſtandard for 10 miles and that theſe meaſures of a foot according to a ſtandard were all made in braſſe by the ſame Workman: and their chaines exactly fitted thereto: and that the meaſure bee taken not along the High-wayes, but by ſide ſtations where Steeples and other places eminent and of note may bee ſeene. If the two townes of the obſervations were *London* and *Edenborough*, it would be preciſeneſſe ſufficient: nay if they were but *London* and *Cambridge*, it would yield a greater certainty then any that I know hath yet beene uſed. This I ſay were an excellent work, and worthy the heroicall magnificence of ſome great man: and yet not of any very chargeable performance: but it would bring a marveilous light and furtherance to Navigation and unto all Aſtronomie.

In the meane time till it ſhall pleaſe God to ſtirre up ſome truely noble ſpirit for the effecting thereof, I will make bold to propoſe a way, which any ingenious ſtudent, whoſe ſight both of his eyes and underſtanding is

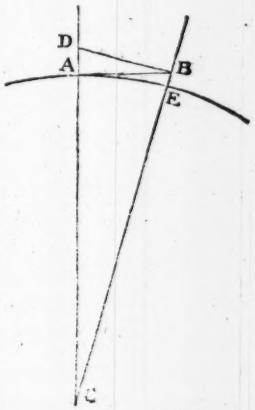
quicke

quicke and perſpicacious, may himſelfe privately *with much facility* practiſe: the reaſon whereof conſiſteth upon theſe *three principles.*

The I. is, *that if with a levelling Inſtrument ſet up in any place parallel to the Horizon a man take a true levell unto another place: the viſuall line by which he levelled, ſhall be a tangent to ſuch an arch of a great circle on the earth, as is contained betweene the ſtation and the marke: Becauſe that the viſuall line, together with the two lines imagined, out of the center of the earth, doe include a right angled Triangle, having the right angle at the levell.*

The II. is, *that if the ſame inſtrument be ſet juſt even with the former mark, and you levell backward to the former ſtation, this laſt viſuall line ſhall overſhoot the former place of the Inſtrument: and ſhall incloſe a new and greater right-angled triangle, having the right angle at the ſecond ſtation.*

The III. is, *that the former of the two viſuall lines ſhall cut this latter and greater right-angled triangle into two right angled triangles like to it ſelf and one like to the other: by the 8 prop: of the 6 book of Euclide.* As in the ſcheme, the center of the earth is C, the firſt place or ſtation of the levelling inſtrument is A, and the viſuall line thence is A B to the marke B, which is alſo the place of the Inſtrumẽt in the ſecond ſtation, from whence the viſuall line backward is B D, over-reaching the firſt place A. Here are 3 like right-angled triangles, namely the greateſt C B D, cut into two other C A B, and BAD, with the line AB. Wherfore A B. AD :: AC. AB: that is; *as the diſtance be-*



between the two Stations (for by reaſon of the vaſt greatneſſe of the earth, and the exceeding ſmall diſtance betweene the two ſtations in compariſon thereof, the viſuall line AB ſhall be the ſame with the ground line AE) *is to the over-ſhooting of the ſecond line of levell: ſo is the Radius to the tangent of the arch* AE, *intercepted betweene the two ſtations.* The quantity of which arch being ſought out in the Canon of tangents, either in *ſexageſime* or *Decimall parts of a degree*, ſay againe, *As the ſame arch in ſexageſime or Decimall parts of a degree is unto a degree in the like parts; ſo is the diſtance betweene the two ſtations in feet, to the number of feet anſwering to a degree upon the earth.* As for example, ſuppoſe the diſtance between the two ſtations to be 528 feet, which is the tenth part of a mile: and that the ſecond line of levell over-ſhooteth the former $\frac{138}{10000}$ of a foote: or $0,\underline{0138}$, which you ſhall finde will bee neere about the matter. Say,

528 . $0,\underline{0138}$ :: 10,0000,0 . 2,61 : *the tangent of the arch* Min, $0,\underline{09}+$

Say againe,

$0,\underline{09}+$. 60 :: 528 . 351120 ; *the number of feet anſwering to a degree upon the earth.*

Thus have I ſet downe the rule, and illuſtrated it with an example. But in the practiſe (by reaſon of the weakeneſſe of our ſight, not able to diſcerne a thing diſtinctly at any great diſtance, we are conſtrained to take but ſhort ſtations, whereby the over-ſhooting of the ſecond line of levell above the firſt is but very ſmall) there is required great preciſeneſſe. For the performance whereof it will not bee amiſſe to ſet downe ſome *directions*, both *concerning the Inſtrument, place, and time.*

The *levelling Inſtrument* to be uſed in this worke, I would not have to bee either with *a channell for water*;

nor

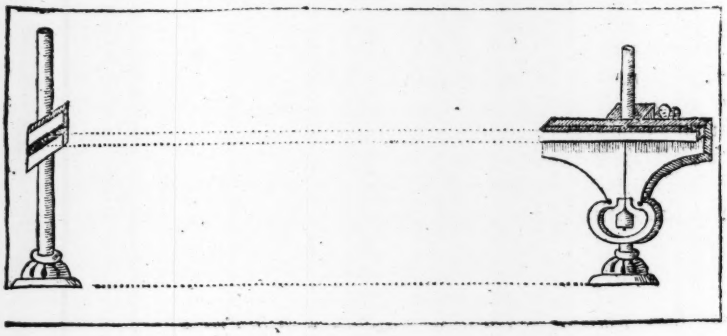
nor with *ſights*. For the *water*, beſides that it *doth continually exhale vapours*, hath a certaine *tenacity*, whereby *to avoyd any dryneſſe* neere to it, it will rather *collect it ſelfe*, and *ſtand in a heape*, then mixe with its enemy: and contrariwiſe very gladly *diffuſeth it ſelfe in purſuit of any moyſture*. And as for *Sights*, if the *ſight-hole be very ſmall, it hindreth our ſeeing*: if any whit *large, it admitteth too many viſive rayes*; which dilating themſelves cannot fixe on *the true and individuall point of the object*. But I would have it onely with *a ledge*, one inch thicke, and three inches broad: and ſo broad alſo I would have *a blacke ſtroke to be in a ſquare white board*, for the *marke to levell at*, that having ſet the ledge of the Inſtrument by the *plumbe-line* parallel to the Horizon in one ſtation, you ſtanding aloofe off, and guiding your eye along the *two edges of the ledge*, and your companion at the other ſtation rayſing up or letting downe the *marke-bourd*, as you ſhall direct him, you may ſee the upper line of the blacke ſtroke levell with the upper edge, and the lower line levell with the lower edge.

The *place for the tryall of this experiment*, I would have to be *a plaine field*, wherein you are to have for your uſe ready meaſured out by the foot, directly *Eaſt* and *Weſt*, ſuch a diſtance, as you can diſcerne diſtinctly thereat: which to a good and perfect ſight may be 1000 feet, or to an indifferent ſight 528 *feet, which is the tenth part of a mile*. And at both ends of that diſtance (which are to be your ſtations) the *ground to be handſomely plained and beaten*, for the more exact ſetting up of your Inſtruments thereon.

The *time for making your obſervation* I would have about *Midſummer, in a ſeaſonable, conſtant, drye, and calme weather*: when, having ſet up your levelling Inſtrument in the *Eaſterne ſtation*, you may take your firſt

levell about *eleven a clocke in the forenoone.* Which being done, you may remove your Inſtrument to the *Weſterne ſtation*, and about *one a clocke in the afternoone* (when the Sun is gone ſo farre paſt the Meridian) take your backe levell.

Theſe are the moſt *neceſſary* and *accurate cautions* that I can deviſe: and all little enough for ſo curious and ſubtill an inquiry. I have alſo here ſet downe *the formes of the levelling Inſtrument and of the marke.*



CHAP.

Chap. IIII.

The manner how to measure the Ships way; *or how many degrees, and parts of a degree, either centesimes, or sexagesimes, the Ship moveth in one houre; or in any space of time assigned. And also of certaine necessary* reductions.

WEe shall therefore come neere the matter if wee take *miles* $66\frac{1}{4}$, that is 349800 *feete to answer to a degree upon the earth.*

Now becauſe the *measure of the Ships motion or way is observed by the watch-glasse and Log-line:* let us for brevity sake call *the number of seconds* (whereof there are 3600 in an houre) *which the Watch-glasse runneth,* by the letter G: and the *number of feet vered in the Logg-line while the glasse is running*, by the letter F. Which grounds being thus layed, wee may find out *a rule to know how many hundreth parts of a degree the Ship sayleth in one houre*; after this manner.

Say $G . F :: 3600 . \frac{3600F}{G}$: *ſo many feet gone in an houre*

Say againe $349800 . 100 :: \frac{3600F}{G} . \frac{360000F}{349800G}$:

Or by *reduction* into parts having the Denominator one

Vnite $\frac{1\frac{92}{100}F}{G}$: which are ſo many *centesimes of a degree gone in an houre.*

Hence ariseth this *generall rule for Centesimes.*

As the number of ſeconds in the Watch-glaſſe, is
to the number of feet vered in the Log-line:
So is $1^{\circ}.9$.
to the number of hundreth parts of a degree,
which the Ship runneth in one whole houre.

But to know *how many minutes of a degree the Ship ſayleth in one houre:* Say againe

$349800 . 60 :: \frac{3600F}{G} . \frac{216000F}{349100G}$: Or by *reduction* into parts having the Denominator one Vnite $\frac{0,6175F}{G}$: which are ſo many *ſexageſimes of a degree gon in an houre.*

Hence alſo ariſeth this *generall rule for ſexageſimes.*

As the number of ſeconds in the watch-glaſſe, is
to the number of feet vered in the Log-line:
So is $0,6.175$,
to the number of minutes of a degree ſayled in
one houre.

Theſe two numbers $1,^{\circ}.9$ and $0,6.175$ (or whether of them you meane to follow) being of moſt frequent, and indeed continuall uſe, it were fit to note in the fourth circle of your Inſtrument with ſome apparant marke: that you may not be ſtill ſearching them out, when you have occaſion to uſe either of them.

And after this very manner you may find a generall rule for any other number of feet contained in a degree upon earth, both for the Decimall parts of a degree, and alſo for the Sexageſimes wherein onely the third termes in every of the ſecond proportions will bee changed.

Becauſe

Becauſe the *true finding out of the way, which the Ship maketh in an houre, eſtimated in the parts of a degree, is the maine ground and principle, by which the place of her being both for longitude and latitude is argued and computed:* I will ſet downe the *practice* thereof at large *in two Examples*: the firſt for *centeſimes of a degree*: and the ſecond for *ſexageſimes:*

Example I. Suppoſe the Watch-glaſſe to containe 40 ſec: and that in the running out thereof the Ship hath gone 175 feete by the Log-line. The rule is, As 40. to 175: ſo is 1,092, to the number of hundreth parts of a degree ſought. Set therefore the antecedent arme of the Index on 40 in the fourth circle, taking the figured diviſions 1, 2, 3, &c. for ſo many tens: and open the other arme unto 175, taking the ſame diviſions for ſo many hundreds: the diſtance betweene the armes will be above halfe that circle. Then remove the antecedent arme unto the third terme 1,092, taking the ſame diviſions for ſo many unites: and the conſequent arme ſhall point at 45, which ſhall be 4 centeſimes and a halfe, or 45 thouſanth parts of a degree, (*viz*) degr: 0,045, in the ſame circuit of that circle: becauſe the diſtance from 40 to 175 outreacheth not the line of 1. Wherefore the Ship at that ſwiftneſſe ſhall goe in an houre degr: 0,045. Which in ſexageſimes will be found to be Min: 2,7.

Example II. Suppoſe the ſame watch-glaſſe of 40 ſec: and that in the running out thereof the Ship hath gone 512 feet. The rule is, As 40 is to 512: ſo is 0,6125, to the number of ſexageſimes or minutes of a degree ſought. Set therefore the anteced nt arme at 40, and the other at 512: the diſtance betweene them exceedeth one whole circuit. Then remove the antecedent arme to the third terme 0,6125: and the conſequent arme ſhall point out 7902: which becauſe the diſtance exceeded

one

one circuit ſhall bee Min: 71902. Which in centeſimes would have beene degr: 01317.

The *proportion of the Ships ſayling for one houre* being thus given either in centeſimes or ſexageſimes of a degree: multiply the ſame by the whole time of the continuance at the ſame ſwiftneſſe reckoned in houres and Decimall parts of houres: and the product ſhall give the whole *way the Ship hath made*, either in degrees or minutes accordingly. As for Example; If the Ship ſayling after degr: 01045 in an houre, continue ſo for Ho. 29, Min: 37, that is Ho: 29617: Multiply 29617 by 01045, and the product ſhall bee degr: 11333, the whole way that the Ship hath made. Or if the Ship for ſo long continuance hath ſayled after Min: 217 in an houre: Multiply 29617 by 217 and you ſhall have Min: 719966, which being diuided by 60, will give degr: 11333, as before.

Now follow *certaine reductions*, which are of frequent uſe. I. *To convert degrees or houres into Minutes*, is to multiply them by 60. And *to convert them into ſeconds*, is to multiy them by 3600. And contrariwiſe.

II. To *rednce minutes into degrees or houres*, is to divide the minutes by 60. And *to reduce ſeconds into degrees or houres*, is to divide them by 3600.

III. To *convert minutes of degrees or houres into centeſimes* or hundreth parts: Say, As 60, is to 100: ſo is the number of minutes, to the number of hundreth parts. And,

IIII. To *reduce centeſimes of degrees or houres into minutes*: Say, As 100, is to 60: ſo is the number of centeſimes or hundreth parts, to the number of minutes.

CHAP.

CHAP. V.

The diuiſion of ſayling into circular and ſpirall. Two fundamentall theorems. Of ſayling by one of the foure Cardinall Rumbes: and certaine Queſtions belonging thereto.

THe *motion of the Ship upon a Rumbe is either circular, or winding with a kind of ſpirall line.* If the ſhip ſaile upon one of the *foure cardinall points*, it deſcribeth a circle: which is either a great circle or leſſer, according as the circle of the heavens is, under which it moveth. For if the Ship ſaileth directly *North or South* under ſome *Meridian*, or directly *Eaſt or Weſt* under the *Æquinoctiall*, it deſcribeth by the motion thereof an arch of a great circle. But if it ſaile directly *Eaſt or Weſt* wide of the *Æquinoctiall* on either ſide, it deſcribeth a leſſer circle, according as the parallel in the heavens is, under which it moveth.

All great circles are equall one to another, and have equall degrees: but the parallels are greater or leſſer one then another; and conſequently have greater or leſſer degrees, as every one is neerer or farther diſtant from the Æquinoctiall. And becauſe in computing the motion of the ſhip we ſhall have continuall occaſion to ſpeake of *degrees both of the greater and leſſer circles*, let this be advertiſed, that as oft as I ſhall mention *Iuſt Degrees*, I underſtand the meaſure of ſo many degrees of a great circle; elſe ſpeaking of leſſer degrees, I call them *proper degrees of ſuch a parallel.*

Theſe two proportions following are the *fundamentall*

tall Theoremes for the computation of the motion of the ſhip: and are therefore faithfully to bee imprinted in our memory. The ſecond is but the converſe of the firſt: and are ſo familiar, that they ſhall neede no demonſtration.

Theor. I. As the *Radius*, is
to the ſine of the complement of the parallel:
So is an arch of the Æquinoctiall in *Iuſt Degrees*,
to the number of *Iuſt Degrees* contained in a like arch of the ſame parallel.

Theor. II. As the ſine of the complement of the parallel, is to the Radius: Or
As the Radius, is
to the ſecant of the parallel:
So is the number of *Iuſt Degrees* contained in an arch of the ſame parallel,
to a like arch of the Æquinoctiall.

If *a Ship ſaile under a Meridian*, that is upon the *North* or *South Rumbe*, it varyeth not the *longitude* at all: but onely changeth the *Latitude*: and that juſt ſo much as the number of degrees it hath runne in that whole time amounteth unto, which number is to be added to the latitude of the place, where the account began, if you have ſayled from the Æquinoctiall-ward towards either Pole: Or elſe to be ſubducted out of the latitude of that place, if you have ſayled towards the Æquinoctiall.

Againe if *the Ship ſayle under the Æquinoctiall upon the very line it ſelfe Eaſtward or Weſtward*: it varieth not the *Latitude* at all: but only changeth the *Longitude*: and that juſt ſo much as the number of degrees it hath runne in that whole time amounteth unto. Which number is to be added to, or ſubducted from the longitude of

of the place wherein you beganne your account, according as you have ſayled *Eaſt* or *Weſt*.

And thirdly *if the Ship ſayle directly Eaſt or Weſt under any parallel circle*, that is upon the Eaſt or Weſt Rumbe, be it in the Northerne or Southern Hemiſphære, it there alſo changeth not the *Latitude* at all, but only the *Longitude*: yet not according to the number of *Iuſt Degrees* it hath gone, as under the Æquinoctiall: but more then ſo many, according as the proportion is betweene that parallel and the Æquinoctiall. For the leſſer every parallel is, the greater muſt needes bee the difference of the Longitude in ſayling ſo many *Iuſt Degrees* under it.

Queſt: I. By the *way of a Ship upon a parallel* being given in *Iuſt Degrees*, to finde how many degrees the *Longitude* is varyed.

This is done at one operation by *Theor*: I.

As the *ſine of the complement of the parallel*, is
to the *Radius*:
So is the *way of the ſhip upon that parallel in iuſt degrees*,
to the degrees of the *difference of longitude*.

An *Example*. A ſhip making her courſe upon the parallel diſtant from the Æquinoctiall degr: 51, min: 32, by the eſtimation of the way hath ſayled 91 in *Iuſt degrees*: how many *proper degrees* of that parallel hath ſhee gone?

The complement of 51°, 32′ is 38°, 28′, the ſine whereof is 62206. Say therefore.

s 38°, 28′	.	Rad	::	91	.	151 111 +.
62206		100000				

The *difference of longitude ſought is degr:* 15|111+: Which arch ſo found is to bee added to, or ſubducted from the longitude of the place where you beganne your account, according as you have ſayled either Eaſt or Weſt.

Queſt: II. How many Engliſh miles change one degree of longitude in going Eaſtward or Weſtward at the elevation of the Pole degr: 51, min: 32.

It was ſuppoſed in the beginning of Chapt: IIII, that miles $66\frac{1}{4}$ doe anſwer to one *degree* of a great circle upon the earth.

The complement of 51°, 32′ is 38°, 28.

Say therefore by Theor: I.

Rad . ſ 38°, 28′ :: 66,25 . 41|211
100000 62206

Wherefore miles 41,211 make a degree on the parallel 51°, 32.

Keepe this number 41|211 in mind for the reſolving of the two queſtions following.

Queſt: III. There are two places having the ſame *latitude* of degr: 51, min: 32: and the *difference* of their *longitudes* is degr: 15|111+: How many *miles* are they *diſtant by the parallel?*

Firſt find out the number of miles anſwering to one degree in the parallel 51°, 32, by *Queſt*: II. which you ſhall find 41|211. Then multiply the ſame by the degrees of the difference of longitude 15|111+: thus,

1 . 41|211 :: 15|111+ . 622,74.

Their diſtance is miles 622,74.

Queſt.

Queſt: IIII. There are two places having the ſame *latitude* of degr: 51, min: 32: and they are *diſtant by the parallel* miles 622,740: how many degrees are they diſtant in *longitude?*

Firſt find out the number of miles anſwering to one degree in the parallel 51°, 32', by *Queſt:* II. Which you ſhall finde 41,211. Then by the ſame number found divide the ſum of the miles given, that is 622,74: thus,

41,211 . 1 :: 622,74 . 15,1114 .

The diſtance of longitude is degrees 15,1114 .

Chap. VI.

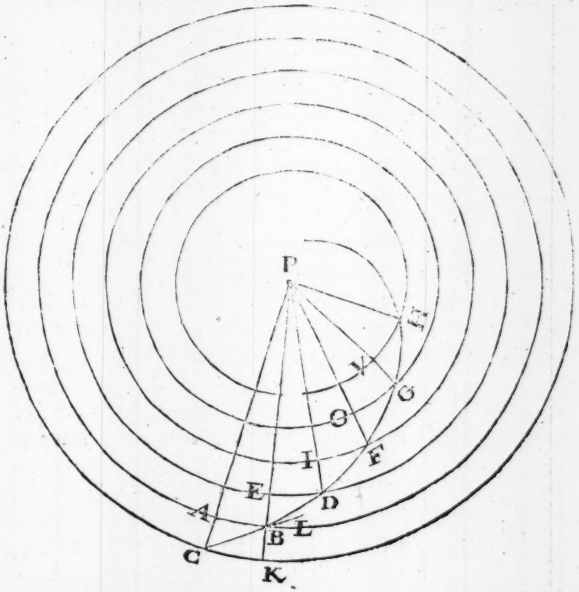
Of the oblique Rumbes *betweene the Meridian, and that of East and West; what they are, and how composed: of finding out certaine* fundamentall Theoremes for oblique sayling.

THat *circular sayling* upon any one of those *foure cardinall points*, whether it bee a *great circle*, or *a parallel*, hath (as wee have seene) no great difficulty in understanding or computing: so that you bee sure of the *true measure of the ships way*: because that therein either only the *latitude*, or only the *longitude* is altered.

But there is greater difficultie in *oblique sayling when the Ship runneth upon some Rumbe between any of the foure cardinall points, making an oblique angle with the Meridian*: because therein the ship continually changeth both *latitude* and *longitude*, And the difficulty is so much the greater by how much the voyage is more distant from the Æquinoctial towards either Pole: and upon *a Rumb* more remote from the Meridian. For neere the Æquinoctiall, where the Meridians are almost parallel; and in those Rumbes which are neere the Meridian, where the longi ude is but little altered; there is no such lubricity and propenseneſſe to erre.

In this kind of *oblique sayling*, the ship is so directed by the Compasse, and guided by the helme, that the *line of the ships length* is every where kept firmely in one and the same angle with the Meridian, according to the distance of that Rumbe from the North and South line. And because the *Compasse is as it were a moveable Horizon*: and the lines of direction thereupon are the *intersections*

ſections of Azumiths or verticle circles with the ſame Horizontall plaine, dividing it into ſo many parts, which are called *Rumbes:* it commeth to paſſe that in ſuch oblique ſayling towards the apparent pole, the place whereunto the Compaſſe leadeth is evermore betweene the parallel through the place wherein you are, and the Pole. Wherefore the *line of the Ships oblique courſe is a helix or ſpirall line, approaching neerer and neerer to the Pole, but never falling into it.* As in the Scheme, ſuppoſe the center of the circle P to be the Pole of the world; and all the concentric circles to be parallels deſcribed at equall diſtance one from the other; and the ſtreight lines



out of the Pole, PAC, PEB, PID, POF, &c. interſecting thoſe parallels in the points C, B, D, F, &c. to be Meridi-

ans

ans: ſo that all the ſegments of the Meridians CA, BE, DI, &c. be equall: and all the ſegments of the parallels KC, BA, DE, &c. bee of equall length, though not of equall degrees; every one of thoſe arches containing foure or more degrees, according as every circle is greater or leſſe then another. Suppoſe alſo that the ſhip keeping a juſt North-Eaſt courſe deſcribeth the crooked line CBDF, which therefore muſt needes be the North-Eaſt Rumbe: and in the continuation of it doth approach unto the Pole neerer and neerer: but can never fall into the Pole: becauſe it ſtill keepeth the ſame diſtance upon the Compaſſe betweene the Meridian, and the parallel in which it is, and maketh with the Meridian an angle of 45 degrees.

Theſe Helices or ſpirall lines (which are the oblique Rumbes) *ought to conſiſt of moſt minute and inſenſible, yea indiviſible parts*: for if they be any whit great, the account of the Ships motion will be confounded, and carryed downe from the true place whither the ſhip is gone, towards the Æquinoctiall: neither can you returne by the Rumbe you came. For imagine in the former Scheme two Meridians PAC, and PBK, and that AB and CK be like ſegments of two parallels, ſo that ABCK ſhall bee a kind of ſphæricall right-angled quadrangle: draw therein diagoniall-wiſe the arch of a great circle CBL, in which the ſhip is ſuppoſed to have gone from C to B: firſt the outward angle PBL being (as may eaſily be demonſtrated) greater then the inward angle ACB, ſheweth that you are fallen from your Rumbe into another point; and had needs to beare up the Ship againe into the Rumbe BD, making with the Meridian an angle PBD equall to that other ACB. Againe, the diagoniall arch CB cutteth the quadrangle into two triangles unequall one to the other: for though in both the ſides AC and BK (which we will call the *cathets*)

catheti) be equall, and the hypotenuſa CB be the ſame: yet the baſes AB & CK, and likewiſe their angles, are unequall: yea though the diſtance of the parallels AC and BK be but one ſcruple of a degree. But yet the leſſe you take the diſtance of the parallels, that inequality will alſo bee the leſſe. So that if by any artifice it may bee brought about that the arch AC be not one minute of a degree, which on the face of the earth anſwereth to above an Engliſh mile, but the hundred-thouſanth, or if need bee the millioneth part of a minute, ſcarce exceeding one fifteenth part of an inch (which thing by the helpe of God the giver of all light I have diſcovered, and am able to performe in tables unto the Radius 10000000, yet nothing at all differing either in their forme or manner of working from thoſe that are now commonly in uſe) all that inequality will be taken away, and thoſe moſt ſmall triangles will indeed, and unto all uſe, become plaine rectangled triangles: and *the ſpirall line of the ſhips courſe be recalled to a preciſe exactneſſe.* By what artifice this is done, together with other ſecrets of that nature, I may peradventure hereafter be induced to declare; if ſo be I ſhall firſt ſee the practiſers of this moſt noble and uſefull ſcience (which is as it were the band and tye of moſt diſjunct countries, and the conſociation of nations fartheſt remote) willingly to relinquiſh their inveterate errours, and to uſe thankfully and conſcionably, without envy and ſelfe-conceited ſtubborneneſſe, ſuch light and helpes as the due and mature ſtudie of true art ſhall afford.

In the meane time we will here make uſe of the *ordinary canon of the Meridian divided according to Mercator:* which I have therefore ſet upon the *ſixth and ſeventh Circles of this Inſtrument*, unto 70 degrees: as hath beene before ſhewed in the ſecond part of the firſt Chapter.

And firſt out of the inſpection of the Rumb in the laſt diagram compacted of the *hypotenuſæ* of an infinite number of thoſe minute rightangled triangles, I wil in certain Theoremes demonſtrate the *ground of oblique ſayling*: And then in the next Chapter apply the ſame foundations to the anſwering of the *ſeverall queſtions in Navigation.*

And becauſe thoſe triangles are all ſuppoſed to bee equall (or rather the ſame triangle ſo often multiplyed) let them be alſo noted with the ſame letters A B C, as the loweſt of them is: the *catheti* C A being all on the *Meridians*: and the *baſes* B A being all on the ſeverall parallels: and the *hypotenuſæ* C B are the motion of the ſhip upon the Rumbe.

The *Theoremes* are ſet downe in theſe proportions.

Theor: I. As the Radius, is
to the ſine of the complement of the *Rumbe*:
So is the *way of the Ship in degrees* upon that Rumb,
betweene any two places on the earth,
to the *difference of latitude* betweene thoſe two places.

For R. *ſ.co* C :: BC. CA :: many BC. ſo many CA.

And ſo converſely.

Theor: I I. As the Radius, is
to the ſine of the *Rumbe* from the Meridian:
So is the *way of the ſhip in degrees* upon that rumb, &c.
to the *ſumme of the baſes of all the triangles intercepted betweene the parallels of thoſe two places.*

For R. *ſ* C :: BC. BA :: many BC. ſo many BA.

Theor.

Theor: III. As the Radius, is
to the tangent of the *Rumbe* from the Meridian:
So is the *difference of latitude* between any two places,
to the *ſumme of the baſes of all the triangles intercepted, &c.*

For R . *t* C :: CA . BA :: many CA . ſo many BA.

Theor: IIII As the Radius, is
to the *ſumme of the ſecants of all the parallels* betweene any two places upon the earth:
So is the *baſe of one of thoſe triangles,*
to the *difference of longitude* between thoſe two places.

For by *Theoreme* II, *Chap*: 5,

R . ſec : parall :: baſe AB . diff: of long : in baſe BA :: many ſec: parall . diff : of long : in ſo many baſes BA.

Againe becauſe by the laſt *Theoreme*

R . ſum : ſec . parall :: BA . diff: long: inBA.

and by *Theor*: III. R . *t* C :: CA . BA .

and becauſe that CA is but 1, be it *ſexageſime or centeſime, &c.* therefore by compoſition of thoſe two proportions ariſeth,

Theor: V. As the quadrat of the Radius, is
to the *ſumme of the ſecants of all the parallels* betweene any two places upon the earth:
Or, As the Radius, is
to the *ſumme of the ſecants of all the parallels* betweene any two places, divided by the Radius:
So is the tangent of the *Rumb* from the Meridian,
to the *difference of longitude* betweene thoſe two places.

CHAP. VII.

Of the ſeverall queſtions which are incident unto oblique ſayling.

IT is needfull to bee advertiſed: Firſt, that in working the queſtions following upon the Inſtrument, the *degrees of the ſhips way* (found out by Chapt: IIII.) and of the *differences both of latitude and longitude*, and alſo the *ſumme of the ſecants of parallels*, are all to be taken on the *fourth circle*, after the manner of abſolute numbers: for which cauſe they are ſtill to be ſet downe in *degrees and Decimall parts of degrees*. But the *Sines* and *Tangents* are to be accounted *in their owne circles*. That heereafter wee may not neede evermore to bee telling unto what circle every number or terme doeth belong,

And ſecondly, that if you pleaſe *to worke theſe queſtions with your pen*: you may doe it by *the tables for the diviſion of the Meridian line according to Mercator*: Which tables are nothing elſe but a perpetuall addition of ſecants. And are to be found both in Maſter *Wrights Errours of Navigation*, and in *Willibrordus Snellius his Tiphys Batavus*, for every minute: and in Maſter *Gunters Booke* for every tenth part of a degree. Which laſt for more readineſſe ſake I doe herein make uſe of.

But in uſing the tables of Maſter *Wright* or *Snellius*, you muſt reckon the *latitudes in degrees and minutes*; *with Decimalls of Minutes*, and not in *Decimalls of Degrees*.

In

In the examples I have ſet downe the numbers ſo, that you may worke them either by the Inſtrument, or with the pen. The manner of working the Decimall parts with the penne you ſhall find in my *Clavis Mathematica, Chapt.* 1, 2, 3, 4, 6. But by the Inſtrument, in the firſt Chapter of this preſent tractate at large: which I could wiſh were dilligently ſtudied and practiſed.

And now I come to the Queſtions.

QVEST. I. *By the Rumbe and way of the Ship given, to find the difference of latitude betweene two places.*

This is done at one operation by *Theor:* I. in the former Chapter.

As the Radius, is
to the ſine of the complement of the Rumbe:
So is the way of the ſhip in degrees upon that rumb,
betweene any two places on the earth,
to the difference of latitude betweene thoſe two places.

An *Example.* A Ship beginning her courſe in the latitude of degr: 50,7, that is 50°, 42', hath ſayled on the *NWbN* Rumbe degr: 9,36: into what latitude is ſhe come?

Here the angle of inclination which the *NWbN* Rumbe maketh with the Meridian is (by Chap: III.) 33°, 45': the complement of which is 56°, 15': and the ſine thereof 83147. Say therefore,

Rad. 100000 . ſ 56°, 15' 83147 :: 9,36 . 7,782 ; *the difference of latitude*

which being added to the Latitude 50 17 given (becauſe the greater latitude is ſought) giveth degrees 58 1482. that is, 58°, 29′.

But if the leſſer latitude had beene ſought: the ſaid difference ſhould have bene ſubducted out of the latitude given. And if the difference of latitude found (the Ship ſayling toward the Æquinoctiall) chance to exceed the latitude given, ſubduct the latitude giuen out of the ſaid difference found: and the remaines ſhall bee the ſecond latitude, but in the contrary Hemiſphære. For if the two latitudes be in the contrary hemiſphæres, the ſumme of both is the difference betweene them.

QVEST. II. *By the way of the Ship and the difference of latitude betweene two places given, to finde the Rumbe leading from one place to the other.*

This is done alſo at one operation by the ſaid *Theoreme* I.

As the way of the Ship in degrees upon the Rumbe ſought betweene any two places, is
to the difference of latitude beween thoſe two places:
So is the Radius,
to the ſine of the complement of the Rumbe ſought.

An Example. A Ship beginning her courſe at the latitude of degrees 50 17, that is 50°, 42′, hath ſayled up to the latitude of degrees 58 1482, that is 58°, 29′: in which ſpace it hath gone degrees 9 136 upon one Rumbe: What Rumb was it that ſhe followed?

Here the difference of latitude is degrees 7 1782. ſay,

9 136 . 7 1782 :: Rad . s 56°, 15′ :
100000 83147

the

the complement of which arch found is 33°, 45′: which is the angle of inclination of the Ships course to the Meridian: and is (by Chapt: III.) the *NW b N* Rumbe.

QUEST. III. *By the Rumbe, and difference of latitude betweene two places, to find the quantity of the Ships way in degrees.*

This is done at one operation by the converse of the said *Theor:* I,

As the sine of the complement of the Rumbe, is
to the Radius:
So is the difference of latitude between any two places
to the measure of the Ships way in degrees.

An *Example.* A Ship beginning her course at the latitude of degr: $58\frac{14}{82}$, hath sayled upon the *S E b S* Rumbe unto the height of degr: $50\frac{17}{}$: how many degrees hath shee gone upon the Rumbe?

Here the difference betweene the two latitudes given is degr: $7\frac{17}{82}$. And the angle of indination of the *SEbS* Rumbe is 33°, 45′, by Chapt: III: the complement of which is 56°, 15′: and the sine thereof 83147. Say therefore.

s 56°, 15′ . Rad :: $7\frac{17}{82}$. $9\frac{13}{6}$:

which is the *measure of the Ships way* in degrees.

QUEST. IIII. *By the Rumbe, and difference of latitude, to find the difference of longitude.*

This is done at one operation by *Theor* V, in the former Chapter.

As

As the Radius, is
to the tangent of the Rumbe:
So is the ſumme of the ſecants of the parallels betweene any two places, divided by the Radius: as they are ſet downe in the tables,
to the difference of longitude between the Medians of thoſe two places.

An *Example.* A Ship beginning her courſe at the latitude of degr: $38\frac{1}{2}$, ſayleth upon the *WbN* Rumb, unto the latitude of degr: $50\frac{5}{6}$: how many degrees of longitude hath it varyed in that courſe?

Here the angle of inclination of the *WbN* Rumbe with the Meridian is 78°, 45′: the tangent whereof is 502734. And the ſumme of the ſecants for $50\frac{5}{6}$ is $58\frac{691}{1000}$: and the ſumme of the ſecants for $38\frac{1}{2}$ is $41\frac{1392}{1000}$: the difference of which is $17\frac{1299}{1000}$, the ſumme of the ſecants of the parallels betweene thoſe two latitudes: which elſe by the Inſtrument is found out by the ſecond part of the firſt Chapter. Say therefore,

Rad	.	t 78°, 45′	::	$17\frac{1299}{1000}$	. $87\frac{927}{1000}$:
100000		502734			86972

Which is the *difference of longitude betweene the Meridians of the two places.*

But becauſe this queſtion is of excellent and very frequent uſe, it will not be amiſſe to ſet downe at large the manner of working this Example upon the Inſtrument. Thus,

Set one of the armes of the Index upon $38\frac{1}{2}$ in the ſixth circle, and open the other arme unto $50\frac{5}{6}$ in the ſeventh circle, according as hath beene taught in the ſecond part of the firſt Chapter. Then move the arme of the

the Index, which ſtood on 38½ to the line of the 1 : and the other arme at the ſame opening ſhall in the *fifth circle* cut 171299, the ſumme of the ſecants.

Againe ſet one of the armes of the Index upon the line of the Radius, and open the other arme unto the tangent of 78°, 45′. Then move the antecedent arme of the Index, which ſtood at the line of the Radius, unto 171209 in the fourth circle: and the conſequent arme ſhall in the ſame fourth circle cut 871927, which are the degrees of the difference of longitude ſought for.

QVEST. V. *By the latitude and longitude of any two places given, to find what Rumbe leadeth from the one place to the other.*

This is done at one operation by the ſame *Theor:* V.

As the ſumme of the ſecants of the parallels betweene thoſe two places, is
to the *difference of longitude* betweene them :
So is the Radius,
to the tangent of the Rumbe ſought.

An Example. There are two places, the one having the latitude of degr: 50½ : and the other the latitude of 38½. And the difference of longitude betweene their Meridians is degr: 871927. By what Rumbe ſhall a Ship ſayle from one place to the other?

Here the ſumme of the ſecants of the parallels betweene the two latitudes given is 171299, as was found out in the example of *Queſt.* IIII. Say therefore,

171299 . 871927 :: Rad . ↑78°, 45′ :
86972 100000 205734

Which is the *angle of the inclination of the Rumb leading between those two places, with the meridian:* and is therfore (by the third Chapter) the *WbN* or *EbS* Rumb: if the latitudes be on the North side of the Æquinoctiall.

QVEST. VI. *By the Rumbe, and difference of longitude betweene two places, whereof one is given, to find the difference of their latitudes.*

This is done at one operation by the converse of *Th.* V.

As the tangent of the Rumbe, is
to the Radius:
So is the difference of longitude betweene the Meridians of those two places,
to the summe of the secants of the parallels betweene those two places.

An Example. A Ship beginning her course at the latitude of degr: $38\frac{1}{2}$, sayleth upon the *WbN* Rumbe untill it hath changed the longitude degr: $87\frac{19}{27}$: Into what latitude shall she then be come?

Here the angle of inclination of the *WbN* Rumbe with the Meridian is 78°, 45′. Say therefore,

t 78°, 45′ . Rad :: $87\frac{19}{27}$. $171\frac{99}{100}$:

Which is the *summe of the secants of the parallels between the latitude degr:* $38\frac{1}{2}$ *given, and the latitude of the place wherein the Ship is.* Wherefore if unto the summe of the parallels for degr: $38\frac{1}{2}$ found out by the second part of Chap: I. namely $41\frac{39}{100}$, you adde the fourth terme found $171\frac{99}{100}$: the summe $58\frac{69}{100}$ shall bee the *summe of the parallels for the latitude sought:* which by the said second part of Chap: I. you shall finde to bee degrees $50\frac{1}{2}$.

QVEST.

QVEST. VII. *By the Rumbe, and measure of the way of the Ship in degrees, to find the difference of longitude betweene two places, whereof one is given.*

This is done by *two operations.*

The first is, By the Rumbe, and way of the Ship given, to finde the difference of latitude: which is *Queſt.* I.

The ſecond is, By the Rumbe, and difference of latitude given, to finde the difference of longitude, which is *Queſt.* IIII.

An Example. A Ship beginning her courſe in the latitude of degr: 50¼ hath ſayled upon the *WNW* rumb deg: 13⅞: how much hath ſhe changed the longitude?

Here the angle of inclination of the *WNW* Rumbe with the Meridian is 67°, 30′: the compl: of which is 22°,30′ the ſine whereof is 38268. Say firſt by *Queſt.* I.

Rad	.	ſ 22°, 30′	::	13⅞ . 5¹²⁴
100000		38268		

Which is the *difference of latitude betweene the beginning and the place where the Ship is.* Now becauſe the Ship ſayling toward the Pole increaſeth the latitude: Adde degr: 5,¹²⁴ to deg. 50¼ the latitude given: and the ſum deg: 55,⁶⁴ ſhall be the *latitude of the place whither he Ship is come.*

Seeke the ſumme of the ſecants of the parallels for both thoſe places, by the ſecond part of Chap: I, which will bee found to bee 58¹⁵³⁴, and 67¹²⁵⁹: the difference of which two numbers is 8¹⁷²⁵, the ſumme of

the ſecants betweene thoſe parallels. Alſo the tangent of the Rumbe, (*viz.*) of 67°, 30′; is 241421.

Say therefore againe by *Queſt.* IIII.

Rad : *s* 67°, 30′ :: 8,725 . 21,064 21,064
100000 241421

Which arch of degr: 21,064 *is the difference of longitude ſought for.*

QVEST. VIII. *By the difference of latitude, and meaſure of the way of the Ship in degrees: to finde the difference of longitude betweene two places, whereof one is given.*

This alſo done by *two operations.*

The firſt is, By the difference of latitude, and the way, to finde the Rumbe: which is *Queſt:* II.

The ſecond is, By the Rumbe, and difference of latitude, to finde the difference of longitude: which is *Queſt* IIII.

An Example. A Ship beginning her courſe in the latitude of degr: 55,64, hath ſayled degr: 13,7 upon one and the ſame Rumbe, even unto the latitude of degr: 50,4: how many degrees of longitude hath ſhe changed?

Here the difference betweene the two latitudes given degr: 5,24

Say firſt by *Queſt.* II.

13,7 . 5,24 :: Rad . *s* 22°, 30′ :
100000 38268

the

the complement of which arch (viz) 67°, 30′, is *the angle of the Rumbe:* And the tangent thereof is 241421. Seeke alſo the ſumme of the ſecants of the parallels for both thoſe places, by the ſecond part of Chap. I: which will be found to bee 58$\frac{534}{}$, and by 67$\frac{259}{}$: the difference of which two numbers is 8$\frac{725}{}$, the ſumme of the ſecants betweene the parallels.

Say therfore againe by *Queſt.* III,

Rad. t 67°, 30′ :: 8$\frac{725}{}$. 21$\frac{064}{}$
100000 241421

Which *arch of degr:* 21$\frac{064}{}$ *is the difference of longitude ſought for.*

QVEST. IX. *By the differences of latitude and longitude betweene two places given, to finde the meaſure of the way of the Ship in degrees.*

This is alſo done by *two operations.*

The firſt is, By the difference of latitude and longitude to finde the Rumbe leading betweene thoſe two places: which is *Queſt.* V.

The ſecond is, By the Rumb, and difference of latitude, to find the meaſure of the Ships way in degrees: which is *Queſt:* III.

An Example. A Ship beginning her courſe in the latitude of degrees 50$\frac{4}{}$, ſayleth ſtill following one and the ſame Rumbe untill ſhee commeth to the latitude of degr: 55$\frac{64}{}$: in which time ſhe hath changed the longitude degr: 21$\frac{064}{}$: How many degrees hath the Ship gone upon that Rumbe?

Here the ſumme of the ſecants of the parallels for both the places propoſed, by the ſecond part of Chap. I,

will be found to be $58\frac{534}{}$, and $67\frac{1259}{}$: the difference of which two numbers is $8\frac{725}{}$, the ſumme of the ſecants of the parallels betweene thoſe two latitudes.

Say firſt by *Queſt.* V,

$8\frac{725}{}$. $21\frac{1064}{}$ :: Rad . *t* 67°, 30′ :
100000 241421

Which is *the angle of inclination of the Rumbe, with the meridian:* the complement of which is deg: 22, min: 30: the ſine whereof is 38268. And the difference betweene the two latitudes degr : $55\frac{164}{}$, and degr : $50\frac{14}{}$, is degr : $5\frac{124}{}$.

Say therefore againe by *Queſt.* III,

s 22°, 30′ . Rad :: $5\frac{124}{}$. $13\frac{17}{}$:
38268 100000

Which is the *meaſure of the Ships way in degrees.*

QUEST. X. *By the Rumbe, and difference of longitude betweene two places, whereof one is given, to finde the quantity of the way in degrees betweene thoſe places.*

This is alſo done by *two operations* :

The firſt is, By the Rumbe, and difference of longitude, to finde out the difference of latitude: which is *Queſt.* VI.

The ſecond is, By the Rumbe, and difference of latitude, to finde out the meaſure of the way of the Ship in degrees: which is *Queſt.* III.

An Example. A Ship beginning her courſe in the latitude of degr : $55\frac{164}{}$, ſayleth upon the *ESE* Rumbe ſo long

long till it hath changed the longitude degr: 21$\frac{64}{100}$: How many degrees hath the Ship gone upon that Rumbe?

Here the angle of the *ESE* Rumbe with the Meridian is degr: 67, min: 30; the tangent whereof is 241421.

Say first by *Queſt.* VI.

s 67°, 30′ . Rad :: 21$\frac{64}{100}$. 8$\frac{725}{1000}$:

Which *is the ſumme of the ſecants of the parallels betweene the latitude of degr:* 55$\frac{64}{100}$, *and the other latitude ſought.*

Now the ſumme of the ſecants of the parallels for the latitude of degr; 55$\frac{64}{100}$ is 67$\frac{259}{1000}$, by the ſecond part of Chap. I. Out of which number if you ſubduct 8$\frac{725}{1000}$ laſt found (becauſe the courſe is towards the Æquinoctiall) the remaines ſhall bee 58$\frac{534}{1000}$, the ſumme of the ſecants of the parallels for the other latitude of degr: 50$\frac{14}{100}$, by the ſame ſecond part of Chapt. I. So that the difference of the latitudes is degr: 5$\frac{124}{1000}$. And the ſine of 22°. 30′, the complement of the *ESE* Rumbe is 38268.

Say therfore againe by *Queſt.* III.

s 22°, 30′ . Rad :: 5$\frac{124}{1000}$. 13$\frac{317}{1000}$
38268 10 000

Which *is the quantity of the Ships way in degrees.*

QVEST. XI. *By the way of the Ship and the difference of longitude betweene the Meridians of any two places, whereof one is given, to find out the Rumbe leading from one place to the other.*

&

QVEST.

QVEST. XII. *By the way of the Ship, and the difference of longitude betweene the Meridians of any two places, whereof one is given, to finde out the difference of their latitudes: by which the other place may be had.*

These two *Questions*, as they are of little or no use in Navigation; so also they have no direct and immediate solution. But are performed after the manner of the *rule of false position*, by supposing reasonably either a Rumbe, or another latitude: and then according to *Quest.* VII, and *Quest.* VIII, to find the difference of longitudes: which if it chance to fall out to be the same that is given in the *Question*; you have your desire. If not: suppose the second time: And lastly by comparing of both errours argue the truth.

These two *Questions* are not so materiall, that I should spend more time in setting downe Examples thereof. I will leave that worke to the studious practiser.

QVEST. XIII. *If it be required to know the distance upon the Rumb between any two places, the measure of the way being knowne in degrees.* You may multiply that measure of the way in degrees by miles 66|25, which is the number of miles contained in one *Iust degree* upon the earth, as was before assumed in Chapt. IIII.

And thus have I shewed *the use of the Instrument in the solution of all nauticall Questions*: which thing I specially in this small tractate aymed at. Which if it shall give any light and satisfaction to such as are studious in that most noble and usefull art, I have my desire: which indeed onely is, that the society of mankind may bee benefited, and God glorifyed, by every poore ability hee hath beene pleased to bestow upon me. I was also in part minded to have annexed hereunto *certain problemes, how*

how by reaſonable conjecture the courſe of the Ship may bee moſt probably rectifyed, when the reckoning thereof by the Compaſſe and way eſtimated, ſhall bee found to diſagree from the cœleſtiall obſervations: Wherein I ſhould have occaſion to ſpeake of the *currents* or *hidden motions of the Seas, how they are to be obſerved, and how to be conſidered of in computing the motion of the Ship*: And alſo of the *deflexion* (or as I may call it, the *bias*) *of the Ship* bending and wheeling it ſelfe about continually to the one ſide; how, and what allowance may moſt reaſonably be made for it. But becauſe theſe doe not properly belong to the Inſtrument, and are to me onely in ſpeculation (which by reaſon of my want of experience in Nauticall affaires, I cannot ſo well direct and ordaine for practice at ſea) I will for this preſent prætermit, contenting my ſelfe with what hath beene already delivered.

And if the *Maſters of Ships* and *Pilots* will take the paines in the journalls of their voyages dilligently and faithfully to ſet downe in ſeverall columnes, not only the Rumbe they goe on, and the meaſure of the Ships way in degrees, and the obſervations of latitude, and variation of their compaſſe; but alſo their conjectures and reaſons of the correction they make of the aberrations they ſhall find, and the quality or condition of their Ship, and the diverſities and ſeaſons of the windes, and the ſecret motions or agitations of the Seas, when they beginne, and how long they continue, how farre they extend, and with what inequallity; and what elſe they ſhall obſerve at Sea worthy conſideration, and will be pleaſed freely to communicate the ſame with Artiſts, ſuch as are indeed skilfull in the Mathematicks, and lovers and inquirers of the truth: I doubt not but that there ſhall in convenient time be brought to light many neceſſary præcepts, which may tend to the perfecting of navigation, and the help and ſafetie of ſuch, whoſe vocations doe en-

force them to commit their lives and eſtates in the vaſt and wide Ocean to the providence of God: to whom be all prayſe, honour, and glory: And this is

The End.

I have at the requeſt of Maſter Elias Allen, *given way that Maſter* Gunters *Table of the diviſion of the Meridian line after* Mercator, *ſhould be here inſerted, for the uſe of ſuch as will take the paines to enter into a numerary calculation of the former Problemes. The other Tables of naturall Sines and Tangents are every where to be had.*

A Table

M	Gr	Par	M	Gr	Par	M	G	Par	M	Gr	Part	M	Gr	Part
00		0	3	3	001	6	6	011	9	9	037	12	12	088
		100		3	10		9	111		9	138		12	190
		200		3	201		6	212		9	239		12	293
		300		3	301		6	312		9	341		12	395
		400		3	402		6	413		9	442		12	497
		500		3	502		6	514		9	543		12	600
		600		3	602		6	614		9	645		12	702
		700		3	703		6	715		9	746		12	805
		800		3	803		6	816		9	848		12	907
		900		3	903		6	916		9	949		13	010
1	1	000	4	4	003	7	7	017	10	10	051	13	13	112
	1	100		4	103		7	118		10	152		13	215
	1	200		4	204		7	219		10	254		13	318
	1	300		4	304		7	319		10	355		13	421
	1	400		4	404		7	420		10	457		13	523
	1	500		4	504		7	521		10	559		13	626
	1	600		4	605		7	622		10	661		13	729
	1	700		4	705		7	723		10	762		13	832
	1	800		4	805		7	824		10	864		13	935
	1	900		4	905		7	925		10	966		14	038
2	2	000	5	5	006	8	8	026	11	11	068	14	14	141
	2	100		5	106		8	127		11	170		14	244
	2	200		5	207		8	228		11	272		14	347
	2	300		5	307		8	329		11	374		14	450
	2	400		5	408		8	430		11	476		14	553
	2	500		5	508		8	531		11	578		14	656
	2	601		5	609		8	632		11	680		14	760
	2	701		5	709		8	733		11	782		14	863
	2	801		5	810		8	834		11	884		14	967
	2	901		5	910		8	936		11	986		15	070
3	3	001	6	6	011	9	9	037	12	12	083	15	15	174

M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Part	M	Gr	Part
15	15	174	18	18	303	21	21	486	24	24	734	27	28	058
	15	277		18	408		21	593		24	844		28	171
	15	381		18	513		21	701		24	953		28	283
	15	485		18	619		21	808		25	063		28	396
	15	588		18	724		21	915		25	173		28	508
	15	692		18	830		21	023		25	282		28	621
	15	796		18	939		22	130		25	392		28	734
	15	900		19	041		22	238		25	502		28	847
	16	004		19	146		22	345		25	613		28	959
	16	107		19	251		22	453		25	723		29	072
16	16	211	19	19	356	22	22	561	25	25	833	28	29	186
	16	316		19	463		22	669		25	943		29	299
	16	420		19	569		22	777		26	054		29	413
	16	524		19	675		22	885		26	164		29	526
	16	628		19	781		22	993		26	275		29	640
	16	732		19	887		23	101		26	386		29	753
	16	836		19	993		23	210		26	497		29	867
	16	941		20	100		23	318		26	608		29	981
	17	045		20	206		23	427		26	719		30	095
	17	150		20	312		23	535		26	830		30	300
17	17	255	20	20	419	23	23	643	26	26	941	29	30	324
	17	359		20	525		23	752		27	052		30	438
	17	464		20	632		23	861		27	164		30	553
	17	568		20	738		23	970		27	275		30	667
	17	673		20	845		24	079		27	387		30	782
	17	778		20	952		24	188		27	499		30	897
	17	883		21	059		24	297		27	610		31	012
	17	988		21	165		24	406		27	722		31	127
	18	093		21	272		24	515		27	834		31	242
	18	198		21	379		24	624		27	946		31	357
18	18	303	21	21	486	24	24	734	27	28	058	30	31	473

M	Gr	Part	M	Gr	Part	M	Gr	Part	M	Gr	Part	M	Gr	Part
30	31	473	33	34	992	36	38	633	39	42	415	42	46	362
	31	588		35	111		38	757		42	544		46	496
	31	704		35	231		38	880		42	673		46	631
	31	820		35	350		39	004		42	802		46	766
	31	936		35	470		39	129		42	931		46	902
	32	052		35	590		39	253		43	061		47	037
	32	168		35	710		39	377		43	191		47	173
	32	284		35	830		39	502		43	320		47	309
	32	400		35	950		39	627		43	451		47	445
	32	517		36	071		39	752		43	581		47	581
31	32	633	34	36	191	37	39	877	40	43	711	43	47	718
	32	750		36	312		40	002		43	842		47	855
	32	867		36	433		40	128		43	973		47	992
	32	984		36	554		40	253		44	104		48	129
	33	101		36	675		40	379		44	235		48	267
	33	218		36	796		40	505		44	366		48	404
	33	336		36	917		40	631		44	498		48	542
	33	453		37	039		40	757		44	630		48	681
	33	571		37	161		40	884		44	762		48	819
	33	688		37	283		41	011		44	894		48	958
32	33	806	35	37	405	38	41	137	41	45	026	44	49	097
	33	924		37	527		41	264		45	159		49	236
	34	042		37	649		41	392		45	292		49	375
	34	161		37	771		41	519		45	425		49	515
	34	279		37	894		41	646		45	558		49	655
	34	397		38	017		41	774		45	691		49	795
	34	516		38	140		41	902		45	825		49	935
	34	635		38	263		42	030		45	959		50	076
	34	754		38	386		42	158		46	093		50	217
	34	873		38	509		42	287		46	227		50	358
33	34	992	36	38	633	39	42	415	42	46	362	45	50	499

M	Gr	Part	M	Gr	Part	M	Gr	Part	M	Gr	Part	M	Gr	Part
45	50	499	48	54	860	51	59	481	54	64	412	57	69	711
	50	641		55	010		59	640		64	582		69	895
	50	783		55	160		59	800		64	753		70	080
	50	925		55	310		59	950		64	924		70	263
	51	068		55	460		60	120		65	096		70	449
	51	210		55	611		60	280		65	268		70	635
	51	353		55	762		60	441		65	440		70	821
	51	495		55	913		60	602		65	613		71	008
	51	639		56	065		60	763		65	786		71	195
	51	783		56	217		60	925		65	960		71	383
46	51	927	49	56	369	52	61	088	55	66	134	58	71	572
	52	071		56	522		61	250		66	308		71	761
	52	215		56	675		61	413		66	483		71	950
	52	360		56	828		61	577		66	659		72	140
	52	505		56	981		61	740		66	835		72	331
	52	650		57	135		61	904		67	011		72	522
	52	795		57	289		62	069		67	188		72	714
	52	941		57	444		62	234		67	365		73	906
	53	087		57	598		62	399		67	543		73	099
	53	233		57	754		62	564		67	721		73	292
47	53	380	50	57	909	53	62	730	56	67	900	59	73	486
	53	526		58	065		62	897		68	079		73	680
	53	673		58	221		63	063		68	258		73	875
	53	821		58	377		63	231		68	438		74	071
	54	968		58	34		63	398		68	618		74	257
	54	116		58	691		63	566		68	799		74	464
	54	264		58	848		63	734		68	981		74	661
	54	413		59	005		63	903		69	163		74	859
	54	562		59	164		64	072		69	345		75	057
	54	711		59	322		64	242		69	528		75	256
48	54	860	51	59	481	54	64	412	57	69	711	60	75	456

M	Gr	Part	M	Gr	Part	M	Gr	Part	M	Gr.	Part	M	Gr.	Part
60	75	456	63	81	749	66	88	725	69	96	575	72	105	579
	75	656		81	970		88	971		96	854		105	904
	75	857		82	191		89	219		97	135		106	230
	76	059		82	413		89	467		97	418		106	558
	76	261		82	635		89	716		97	701		106	888
	76	464		82	860		89	967		97	986		107	220
	76	667		83	084		90	218		98	272		107	553
	76	871		83	310		90	470		98	560		107	888
	77	076		83	536		90	723		98	849		108	226
	77	281		83	763		90	978		99	139		108	565
61	77	487	64	83	990	67	91	232	70	99	431	73	108	906
	77	694		84	219		91	489		99	724		109	249
	77	901		84	448		91	746		100	018		109	594
	78	109		84	678		92	005		100	314		109	941
	78	317		84	909		92	264		100	612		110	290
	78	526		85	141		92	525		100	910		110	641
	78	736		85	374		92	787		101	211		110	994
	78	947		85	607		93	050		101	513		111	349
	79	158		85	842		93	314		101	816		111	707
	79	370		86	077		93	579		102	121		112	066
62	79	583	65	86	313	68	93	846	71	102	427	74	112	428
	79	796		86	550		94	113		102	735		112	792
	80	010		86	788		94	382		103	044		113	158
	80	225		87	027		94	652		103	356		113	526
	80	441		87	267		94	923		103	668		113	897
	80	657		87	508		95	195		103	983		114	270
	80	874		87	749		95	468		104	229		114	645
	81	091		87	992		95	743		104	616		115	023
	81	310		88	235		96	019		104	936		115	403
	81	529		88	480		96	296		105	257		115	786
63	81	749	66	88	725	69	96	575	72	105	579	75	116	171

M	Gr.	Part	M	Gr.	Part	M	Gr.	Part	M	Gr.	Part	M	Gr.	Part
75	116	171	78	129	075	81	145	650	84	168	947	87	208	705
	116	559		129	558		146	292		169	913		210	649
	116	949		130	045		146	942		170	893		212	668
	117	342		130	536		147	600		171	891		214	745
	117	737		131	031		148	265		172	907		216	909
	118	135		131	530		148	937		173	941		219	158
	118	536		132	034		149	618		174	994		221	498
	118	639		132	542		150	307		176	067		223	938
	119	345		133	055		151	003		177	160		226	486
	119	755		133	572		151	709		178	275		229	153
76	120	166	79	134	094	82	152	423	85	179	411	88	231	950
	120	581		134	620		153	147		180	569		234	891
	121	000		135	151		153	878		181	752		237	991
	121	420		135	687		154	620		182	960		241	268
	121	843		136	228		155	372		184	194		244	744
	122	270		136	775		156	132		185	454		248	445
	122	700		137	326		156	903		186	743		252	402
	123	133		137	883		157	685		188	062		256	652
	123	570		138	445		158	478		189	411		261	243
	124	009		139	012		159	281		190	793		266	235
77	124	452	80	139	585	83	160	096	86	192	210	89	271	705
	124	898		140	164		160	922		193	661		277	753
	125	348		140	748		161	761		195	151		284	517
	125	801		141	339		162	612		196	680		292	191
	126	258		141	936		163	475		198	251		301	058
	126	718		142	538		164	352		199	867		311	563
	127	182		143	147		165	242		201	529		324	455
	127	649		143	763		166	146		203	240		341	166
	128	121		144	385		167	065		205	005		365	039
	128	596		145	014		167	999		206	825		408	011
78	129	075	81	145	650	84	168	947	87	208	705	90	Infinite.	

The Declaration of the two RVLERS for *Calculation*.

THe *Rulers* are ſo framed and compoſed, that they may not only be applyed to the *calculation of Triangles*, and the *reſolution of Arithmetical Queſtions*: but that they may alſo very fitly ſerve for a *Croſſe-ſtaffe* to take the height of the Sunne, or any Starre above the Horizon, and alſo their diſtances. In which regard I call the *longer* of the *two Rulers* the *Staffe*, and the *Shorter* the *Tranſverſarie*. And are in length one to the other almoſt as 3 to 2.

The *Rulers* are juſt foure ſquare, with right angles: and equall in bigneſſe: they are thus divided.

The *Tranſverſarie* at the upper end noted with the letters *S*, *T*, *N*, *E*, on the ſeverall ſides, hath a *pinnicide* or *ſight*: at the lower edge of which ſight is the *line of the Radius*, or *Vnite line*, where the diviſions beginne.

On the left edge of one of the ſides are ſet the *Degrees* from 0 to 33 degrees. And on the right edge of the ſame ſide is ſet the *line of Sines* from 90 to 1 degree.

In the next ſide are ſet *two lines of Tangents*, that on

the right edge goeth upward from 1 to 45 degrees: and that on the left edge goeth downeward from 45 to 89 degrees.

In the third side, on the right edge is set the *line of Numbers*, having these figures in descent, 1, 9, 8, 7, 6, 5, 4, 3, 2, 1, 9, 8, 7, &c.

In the fourth side on the right edge is set the *line of Æquall parts*: And on the left edge are diverse *chords* for the dividing of Circles.

The *Staffe* at the further end of it hath *a socket* with *a pinnicide* or *sight*: at which beginneth the 30 degree, and so goeth on to 90 degrees at the end of the Staffe next your eye: which degrees from 30 to 90 are set on the right edge of one of the sides of the Staffe.

Then applying your *Transversarie* to the *Staffe* with the lower end set to 90, marke on the foure sides of the Staffe *the line of the Radius* or *Vnite*: at which on every left edge must beginne the *single line of Sines, Tangents*, and *Numbers*, the very same which were in the *Transversarie* (that of the Sines being on that side where the degrees are) only the *line of Tangents, and numbers are continued* beyond the line of the Radius, to the further end of the Staffe.

And on the fourth side of the Staffe in the middle are *double divisions*: that on the right hand is *a line of Æquall parts* to 100, reaching the whole length of the Staffe: And on the left hand contiguous to the former, is the *line of Latitudes* or *Elevations of the Pole* unto 70 degrees marked with the leter *L*.

The *degrees* both of the Staffe, and Transversarie, and also

alſo of the Sines and Tangents may bee divided into 6 parts which containe 10 minuts apiece: or rather into 10 parts containing 6 minutes apiece: for ſo they may ſerve alſo for Decimals.

Thus have you on the *two Rulers* the very ſame lines which are in the *Circles of Proportion:* and whatſoever can be done by thoſe Circles, may alſo as well be performed by the two Rulers: and the Rules which have bin here formerly ſet downe for that Inſtrument, may alſo be practiſed upon theſe: ſo that you bee careſull to obſerve in both the different propriety in working. It will not therfore be needfull, to make any new and long diſcourſe, concerning theſe Rulers, but onely toſhew the manner, how they are to be uſed, for the calculation of any proportion given.

In working a Proportion by the Rulers, *hold the Tranſverſary in your left hand, with the end at which the line of the Radius or Vnite line is, from you ward: turning that ſide of the Ruler upward, on which the line of the kind of the firſt terme is, whether it be Number, Sine, or Tangent: and therein ſeeke both the firſt terme, and the other which is homogene to it. Then take the Staffe in your right hand with that ſide upward, in which the line of the kind of the fourth terme ſought for is: and ſeeke in it the terme homogene to the fourth. Apply this to the firſt terme in the Tranſverſarie: and the other homogene terme ſhall in the Staffe ſhew the fourth terme.*

As if you would multiply 355 by 48: Say

1 . 355 :: 48 . 17040 .

For if in the line of Numbers on the Staffe you reckon 355, and apply the ſame to 1 in the line of Numbers on the Tranſverſarie; then ſhall 48 on the Tranſverſarie ſhew 17040 on the Staffe.

Againe if you would divide 17040 by 48 : Say

48 . 1 :: 17040 . 355

For if in the line of Numbers on the Tranſverſarie you reckon 48, and to the ſame apply 1 in the line of Numbers on the Staffe : then ſhall 17040 on the Tranſverſarie ſhew 355 on the Staffe.

The true value of the fourth terme found, may be had by the 5 & 6 ſect : 2 chap : 1 part.

Some Examples of working Proportions wee will borrow out of 3 chap : 1 part

Example I. If 54 elnes of Holland be ſold for 96 ſhillings : for how many ſhillings ſhall 9 elnes be ſold ? the worke ſhall be thus

$54^{el} . 96^{sh} :: 9^{el} . 16^{sh}$;

for if in the line of Numbers on the tranſverſarie you ſeeke the firſt terme 54 elnes, and in that line on the Staffe you ſeeke 96 ſhillings : and apply one to the other : then ſhall 9 elnes ſought out on the Tranſverſarie point out 16 ſhillings on the Staffe.

Example IIII. There is a Tower whoſe height I would meaſure.

I take two ſtations in the ſame right line from the Tower : and at either ſtation having obſerved the height by the ſights of the Staffe, I find the neerer ſtation 28 deg : 7 min : almoſt : and the further ſtation 21 degr : 58 minutes alm oſt : and betweene both the Stations the diſtance was 76 feet.

The

The rule of measuring heights by two stations is contained in these *Theoremes*.

THEOR. *As the difference of the Tangents of the arches cut in either Station, is to the distance betweene the stations: so is the Tangent of the lesser arch, to the neerer distance from the Tower.* Againe

THEOR. *As the Radius is to the Tangent of the greater arch; so is the neerer distance found, to the height.*

And therefore because according to 6 sect: 1 chap: 1 part, by application of the line of Numbers to the line of Tangents (*that is by applying the Vnite line of the Staffe, to the Tangent in the Transversarie, if the arch be lesse then 45 degr: but if the arch exceed 45 degr: by applying the said Vnite line, unto the arch it selfe, or the complement thereof, which in the Transversarie is all one*) the Tangent of 28°, 7′ is 5343, and the tangent of 21°, 58′ is 4033: whose difference is 1310: the Proportions will in the lines of Numbers be thus

First, 1310 . 76 :: 4033 . 234

wherfore 234 feet is the neereſt distance

Second. Radius . tang: 28°, 7′ :: 234 . 125

wherfore 125 feet is the height sought for.

Or else you may resolve it at one operation thus,

THEOR. *As the difference of the tangents of the complements of the arches cut in either station,*
is to the Radius;
So is the distance betweene the stations,
to the altitude.

Becauſe accordingly as was before ſhewed, the tangent of the complement of 28°, 7′ is 18715 : and the tangent of the complement of 21°, 58′ is 24792 : whoſe difference is 6077. the proportions will in the line of Numbers be thus

6077 . 10000 :: 76 . 125 .

And theſe Rules may be alſo applyed to find out the diſtances of objects.

Example V. To find the declination of the Sunne the ninth day of May.

Becauſe upon the ninth day of May the place of the Sunne is in ♉ 29 : which is 59 degrees diſtant from the next Æquinoctiall point. Say in the line of Sines

Radius . ſine 59° :: ſine 23°, 30′ . ſine 19°, 59′ .

And ſo much is the declination ſought for.

If the diſtance of the Sunne from the next Æquinoctial point exceed not degrees 2°, 30′. Breake that arch into minutes, or decimal parts of a degree : and by the lines of Sines and numbers ſay

As the Radius is
to the Sine of 23°, 30′;
So is the diſtance (of the Sunne from the next Æquinoctial point) in minutes, or Decimals,
to the declination in minutes, or Decimals.

As if the declination of the Sunne being in ♍ 27°, 45′ be required : the diſtance of it from the next Æquinoctial point is 2°, 15′ that is minutes 135, ſay therefore

Rad

Rad . ſine 23°,30′ :: 135′ . 53½′ .

which is the declination of the Sunne in that place.

Example VI. To find the right aſcenſion of the Sunne upon the ninth of May.

Becauſe upon the ninth of May the Sunne is 59 degr: diſtant from the next Æquinoctiall point: ſay in the line of Sines on the Tranſverſarie, and the line of tangents on the Staffe

Rad . ſin: compl: 23°,30′ :: tan: 59° . tan: 56°,46′.

which is the Sunnes right aſcenſion upon the ſame day.

Or elſe (becauſe the Radius is the meane proportional betweene the tangent of an arch and its complement) the ſame proportion might have beene thus ſet downe.

t: com: 59° . Rad :: ſin: com: 23°,30′ . t: 56°,46′,

In which manner of propoſure happening onely when there is in the proportion the Radius and two tangents, becauſe the two homogenes of the one kind are both extreame termes, and the two homogenes of the other kind are both middle termes: the tangent is to be turned into the tangent of the complement: and muſt change places with the Radius. As by comparing the two former proportions doth plainely appeare.

Becauſe that the greateſt difficultie of working by theſe Rulers falleth out in the tangents, when the arches are in the ſecond mediety of the Quadrant, it will bee convenient

convenient to set downe *some cases wherein the worke differeth from the ordinary manner.*

Case I. If the foure proportionals being all tangents, the arches of two of the termes given exceed 45 deg: and the arch of the third be lesse then 45 degr: as in this Example

tan: 56° . tan: 31° :: tan: 79° · tan: 64°, 24′:

here the tangent of 31° on the Transversarie being applyed to the tangent of 56° on the Staffe, the tangent of 79° on the Staffe will outreach the Radius or end of the Transversarie. Wherefore to find out the fourth proportional, marke what point of the Staffe, the line of the Radius on the Transversarie doth touch, and to it (turning the Transversarie) set the other end of the line of the Radius, and so shall the tangent 79° in the Staffe give you tangent 64°, 24′ in the Transversarie.

Case II. If the foure proportionals being all tangents, the arches of the three termes given exceed 45 deg. as in this Example

tan: 79° . tan: 56° :: tan: 64°, 24′ . tan: 31°.

turne the Transversarie, and set the tangent of 56° therein to tangent 79° on the Staffe: and because the Radius or end of the Transversarie reacheth not to the tangent 64°, 24′ on the Staffe: to find out the fourth proportinal, marke what point of the Staffe the line of the Radius of the Transversarie doth touch, and to it (turning the Transversarie) set the other end of the line of the Radius, and so the tangent 64°, 24′ in the Staffe, will give you tangent 31° in the Transversarie.

These

Theſe two Cales, being nothing elſe but a ſupplying of the ſhortneſſe of the Tranſverſarie, may ſerve as a rule, and direction for all other workes of the ſame kind.

Concerning the manner of working *by Quadrats* and *Cubes* upon the line of Numbers. And of *duplicated* and *triplicated proportions.*

The difference of a Quadrat from a Quadrat is double the difference between their ſides.

And *the difference of a Cube from a Cube is triple the difference betweene their ſides.*

Example II, *chap: 6, part* I. How many acres of Wood-land meaſured with a Perch of 18 feet, are there in 73 acres of Champane-land meaſured with a Perch of 16$\frac{1}{5}$ feet?

The meaſures given, 18, 16$\frac{1}{5}$ being reduced into the leaſt termes, are as 12 to 11, and the proportion is reciprocall. Say therefore,

$$\square : 12 \;,\; \square : 11 \;::\; 73 \;.\; 61\tfrac{1}{34}$$

Which is thus wrought: In the line of Numbers apply 11 on the Staffe, to 12 on the Tranſverſarie, then ſhall 73 on the Tranſverſarie give 67 — on the Staffe: which 67 — being reckoned on the Tranſverſarie (the Rulers ſtanding as they did) ſhall on the Staffe give 61$\frac{1}{34}$ the number of acres in Wood-land meaſure.

Example III, *chap 6, part* 1. If pounds 0$\frac{1}{43}$ of gun-powder ſuffiſe to charge a Gunne whereof the concave Diameter is ynches 1$\frac{1}{5}$: how many pounds of powder

will suffise to charge a Gunne whose concave Diameter is ynches 7?

The capacities are one to the other as the Cubes of the Diameters. And the proportion is direct. Say therfore

C: 1$\frac{5}{}$. C: 7 :: 0$\frac{143}{}$. 43$\frac{17}{}$

Which is thus wrought: In the lines of Numbers apply 7 on the Staffe, unto 1$\frac{5}{}$ on the Transversarie, then shall 0$\frac{143}{}$ on the Transversarie give 2$\frac{101}{}$— on the Staff: which 2$\frac{101}{}$— being reckoned on the Transversarie (the Rulers standing as they did) shall on the Staffe give 9$\frac{333}{}$: and againe the same 9$\frac{333}{}$ being reckoned on the Transversarie shall on the Staffe give 43$\frac{17}{}$ the quantity of pounds of powder suffising.

Example in 46 pag. of Navigation. A ship beginning her course at the Latitude of deg: 38$\frac{1}{2}$, saileth upon the *Wb N* Rumbe, unto the Latitude of deg: 50$\frac{5}{}$: how many degrees of Longitude hath it varied in that course?

Here the angle of Inclination of the *Wb N* Rumbe with the Meridian is 78°,45′, the tangent whereof is 502734. And by the *double divisions on the fourth side of the Staffe*, the Summe of the Secants for the Latitude of 50,5 is 58$\frac{697}{}$: and the summe of the Secants for the Latitude of 38$\frac{1}{2}$ is 41$\frac{1392}{}$: the difference of which is 17$\frac{1299}{}$; the summe of the Secants of the parallels betweene those two Latitudes. Say therefore.

Rad . tang: 78°,45′ :: 17$\frac{1299}{}$. 86$\frac{1268}{}$

which is the difference of Longitude betweene the Meridians of the two places.

But because this question is of excellent and very frequent

quent uſe, it will not be amiſſe to ſet downe at large the manner of working this Example upon the Rulers.

Looke the two Latitudes 50½, and 38½ given, in the *line of Latitudes or eleuations of the Pole on the fourth ſide of the Staffe:* and either marke what number each of them ſheweth in the *line of Æquall parts* there, which you ſhall find to be 58.691, and 41.392, the difference of which is 17.299, as was before ſaid: or elſe more eaſily, ſet one foot of your Compaſſes on one of the Latitudes given, and open the other foot to the other Latitude given: then keeping that aperture, ſet one of the feet in the beginning of that *line of Æquall parts,* and the other foot ſhall upon the ſame line ſhew the difference of Secants betweene the ſaid two Latitudes given, that is 17.299.

Then in working the Proportion; becauſe the angle of inclination of the Rumbe 78°, 45′ is more then 45 deg: turne that edge of the Tranſverſarie on which the tangents of arches above 45 are, toward the Staffe in your right hand: and to the line of the Radius apply 17.299 ſought out on the line of Numbers on the Staffe: and ſo ſhall tang: 78°, 45′ on the Tranſverſarie: ſhew 86.97 — on the Staffe.

The Vſe of the Croſſe-Staffe.

FOr the more ready uſe of the Croſſe-ſtaffe, you are to remember that *the degrees ſerving for the Croſſe-ſtaffe are placed both on the Staffe and Tranſverſarie, on the ſame ſide on which the line of Sines is.* And that in framing thereof *the Tranſverſarie is to be ſet in the Socket ſo that it may ſtand on the right hand of the Staffe.*

The degrees on the Transversarie are only the first 30. and serve to shew an angle not exceeding 30 degrees.

Yet it would not be unusefull if both the Transversary and Staffe were made somewhat longer that the Transversarie might containe 5 deg: after 30; and the Staffe 5 degrees before 30.

To find an angle lesse then 30 degrees between any two objects.

Place the Socket at 30 deg: on the Staffe; and skrew it fast there: then setting the end of the Staffe to your eye, draw the Transversarie up and downe through the Socket, till you may see with your eye the two objects upon the two sights of your Crosse-staffe: and so shall the degrees cut on the Transversarie shew you the angle of their distance, if it be not above 30 degrees.

To find an angle greater then 30 degrees, between any two objects.

Place the Socket at 30 degrees of the Transversarie, and skrew it fast there: then setting theend of the Staffe to your eye, draw the Socket up and downe along the Staffe till you may see with your eye the two objects upon the two sights of your Crosse-staffe: and so shall the degrees cut on the Staffe, shew the angle of their distance, if it be above 30 degrees.

And thus much, together with that which hath been before taught in *Example* IIII: will bee sufficient for the Vse of the Crossestaffe: especially seeing so many men have already writ ten upon this Argument.

Soli Deo gloria.

FINIS.

The Translator to the Reader.

Gentle Reader, by reason of my absence, whilest this Booke was in the Presse, it is no mervaile though some faults have escaped, which you will be pleased to amend thus.

Pag. 3, *lin.* 1, the *third circle* p. 8, *lin.* ult. so $\frac{44}{350}$ is $0,\underline{125}$. *pag.* 14, *lin.* 14, 2.0413927 *pag.* 15, *lin.* 1, the first terme of a progression *p.* 16, *lin.* 24, $108\frac{133}{+}$; *pag.* 17, *lin.* 17, the antecedent arme *lin.* 28, 4 chap. *pag.* 18, *lin,* 19, terme given from *pag.* 19, *lin.* 11, in the fift circle *pag.* 20, *lin.* 19, lye hid. As in this *lin.* 20, D . rat . mul^{ta} — 1 in R :: α . Z .
lin. 28, and Rat. mul^{ta} in R in α — R in α,
lin. ult. and Rat. mul^{ta} in R — R, in α,
pag. 21, *lin.* 1, *and* Rat mul^{ta} in α — α, in R,

And also in the Æquations pag. 21, 24, 26, *which haue a magnitude æquall to a fraction: the same magnitude together with the note of æquality, ought to be set right against the line that is betweene the Numerator and Denominator of the fraction, as in these,* $\frac{\text{Rat . mul}^{ta} - 1 \text{ in R in } \alpha}{D} = Z$.

And $\frac{ZD}{\alpha} =$ Rat . mul^{ta} in R — R . And so of the rest.

pag. 24, *lin.* 25, $\frac{\text{Rat . mul}^{ta} - 1 \text{ in R in } \alpha}{\text{Rat . mul}^{ta} \text{ in D}}$

pag. 25, *lin.* 20, arme at $71\frac{382}{}$:
lin. 23, (for it is Rat. mul^{ta} — 1 in R in α)
pag 26, *lin.* 16. Ratiocination *pag.* 29, *lin.* 29, number of figures *pag.* 35, *lin.* 5, $61\frac{49}{144}$ *pag.* 36, *lin.* 11, $431\frac{7}{}$. $17\frac{48}{}$. *lin.* 14, :: $17\frac{148}{}$. $3\frac{264}{}$ *pag.* 37, *lin.* 11, $3\frac{14416}{}$, *pag.* 39, *lin.* 15, $339\frac{2928}{}$ *pag.* 41, *lin.* 19, or as 1 is to $1,\underline{0472}$: *pag.* 44, *lin.* 8 is a roofe *lin.* 26, thereof *pag.* 45, *lin.* 10, feet $529\frac{175}{}$. *pag.* 46, *lin* 3, more sides then foure *pag.* 53, *lin,* 10, Cylindrical vessel.

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To find an angle greater then 30 degrees, between any two objects.

Place the Socket at 30 degrees of the Transversarie, and skrew it fast there: then setting theend of the Staffe to your eye, draw the Socket up and downe along the Staffe till you may see with your eye the two objects upon the two sights of your Crosse-staffe: and so shall the degrees cut on the Staffe, shew the angle of their distance, if it be above 30 degrees.

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pag. 53, *lin.* 18, if false, why *lin.*20, error? *pag.* 57, *lin.*14, common opinion is, that at London a Cylindrical *lin.* 28, 16,5 *pag.*74, *lin.*28, the Summe, *pag.*78, *lin* 6, third houses *lin.*28, Adde 90 degrees *pag.*79, *lin.*22, and the 90th degree *pag.*82, *lin.*5, 26 *pag* 94, *lin.*21, a circle, or 90 degrees. *pag.*95, *lin.* 34, angle D be obtuse, *pag.*96, *lin.*4, signe + *lin.*21, $\sqrt{q} : Z + X$: *pag.*100, *lin.* 3, and then the side DC *In the* VIII *diagramme of right-lined Triangles the leter* A *is wanting at the perpendiculer. And in the* VI *the angle* B *ought to have been marked with a little line.* *pag.*113, *lin.*7, the delineation *pag.* 127, *lin.* 1, the sunne goeth not under *pag.* 131, *lin.*19, in the paper *pag.*132, *lin.*10, to the tangent of the arch. *pag.* 134, *lin. ult.* North or South direct inclining: *pag.* 135, *lin.* 1, North or South direct reclining. *lin.*8, either face of the Plaine looketh: *p.*143, *lin.*3, In North reclining and South inclining *pag.* 144, *lin.*19, instrument through the Pole of the Æquinoctial, is one of these three. *pag.*152, *lin.* 30, North dyal declining Eastward 35 degrees

In the Additament of Navigation.

*pag.*2, *lin.*11, *for* valure *reade* value *pag.*6, *lin.*9, & 12, for *signes* reade *sines* *pag.*29, *lin.*20, 21, 24, *for* 45 *read* 48 *lin.*20, *for* an halfe *reade* eight thousanth parts & *lin.* 25, *for* min: 2,7 *reade* min: 2,88 *pag.*45, *lin.* 20 *pag.* 47, *lin.* 9, & 28, *pag* 48, *lin.*17 & 21, *for* 87,927: *read* 86,968 *pag.* 47, *lin. ult. for* 205734, *read* 502734. *pag.*28, *lin.*8, $\frac{216000F}{349800G}$: